SPECTRAL CHARACTERISTICS OF FEMTOSECOND PULSES AT CLASSICAL AND QUANTUM METHODS OF THE DESCRIPTION

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Abstracts

Enormous importance of femtosecond pulses in a modern science and the engineering demands all-round studying of their spectral and time characteristics. Femtosecond pulses represent a special kind of electromagnetic pulse signals, and time and spectral description is being under doubts in known references. In this work results of calculations of supershort radio pulses are presented with number of the periods from one to ten. This calculations have universal sense for any frequencies of harmonious oscillations. Also the line width emission of the concrete femtosecond laser has been found.

I. INTRODUCTION

Tendency to creation of shorter optical pulses was outlined in the beginning of development of laser technology. On the modern stage of development pulses are received by duration from tens femtosecond up to 5–6 fs, such limiting values conform to two-three and to even one period of light oscillation [1]. Development attosecond range has begun. Femtosecond pulses have huge value in a modern science and the engineering. For example, with their help it is possible to study high speed processes, taking place in light reaction with substance, the chemical and photochemical reactions, in plasma physics (Fundamental Research), to create accurate time and length measurement systems. Femtosecond pulses are used for creation of electromagnetic fields with intensity above subatomic. Intensive researches on optical computers creation are conducted. Femtosecond lasers become also a basis of new technology of optical communication which can pass terabytes of information per second.

All above told, makes actual frequency and time characteristics femtosecond pulses researches. Therefore it is very important to research their time and frequency characteristics. It can open new ways of their application.

Experimental measurement of duration femtosecond pulses is challenging because of their small duration. Traditional methods of time measurements with use of photoelectronic devices together with the most high-speed oscillographs provide temporary resolving which is much lower to duration femtosecond pulses. Indirect methods for a finding of duration of ultrashort pulses are known – by Fourier transformation (spectra have been experimentally measured). But they have not got any further progress.

All this shows that theoretical and experimental researches of femtosecond pulses demands further researches. Therefore the objective of the given work is theoretical research of spectra of the ultrashort pulses containing small number of the periods, beginning from one period.

The objective relation between time and frequency characteristics of signals at research femtosecond pulses has been called in a doubt [2, 3]. Whether the answer to a question «so it?» will give theoretical research of spectra of femtosecond pulses with use Fourier transformation (the classical theory).

II. METHODS OF RESEARCH OF TIME-FREQUENCY CHARACTERISTICS OF FEMTOSECOND PULSES

In the literature two forms of representation femtosecond pulses in the form of an analytical signal are known [4]. The first of them:

$$\hat{s}(t) = \hat{A}(t)\exp(i\omega_0 t), \qquad (1)$$

where $\hat{\hat{A}}(t)$ – complex envelope of signal.

The second form of analytic form of femtosecond pulses has the form

$$s(t) = a(t)\cos\omega_0 t \qquad (2)$$

The concrete definition of the factor a(t) is known in the following form

$$s(t) = \mathrm{Re}\,ct\left(\frac{t}{T}\right)\cos\omega_0 t$$
$$\mathrm{Re}\,ct = \begin{cases} 1, t \in T \\ 0, t \notin T \end{cases} \quad (3)$$

Analytic forms of femtosecond pulses demand serious discussion.

It is possible to present any oscillation process in the form

$$s(t) = a(t)\cdot\cos\varphi(t) \quad (4)$$

function a(t) name amplitude (or envelope), φ(t) – phase, $\omega(t) = \dot{\varphi}(t)$ – instantaneous frequency.

Representation of a signal s(t) in the form of factors is ambiguous, because at any function φ(t) always it is possible to satisfy to the equation (4) corresponding choice of function a(t). For a long time it is known that expansion of the function on composed and on factors should be adequate [5].

If the process spectrum is concentrated near to carrier frequency suppose $\varphi(t) = \omega_0 t + \hat{O}(t)$. In this case amplitude a(t) and additional phase Φ(t) change in time essentially more slowly, than cosω_0t or inω_0t that allows to concrete definition of the expression (4):

$$s(t) = a(\mu\omega_0 t)\cdot\cos[\omega_0 t + \hat{O}(\mu\omega_0 t)]. \quad (5)$$

Determinacy of factoring exists, when envelope a(μω$_0$t) and phase Φ(μω$_0$t) signal S (t) are connected and the condition that $\mu \ll 1$ is satisfied. The signal is narrow-band and it is possible to pass to representation (1) [4].

Consideration femtosecond pulses in the form of (2) means that one is chosen from set of every possible representations in the form of product and its physical sense doesn't speak.

Further passage researches multiplier of envelope in various physical systems were conducted. As a result it appears that function $\hat{A}(t)$ (or Rect (t/T)) oscillate faster, than a phase factor. All it contradicts concept envelope. And the question on time-and-frequency characteristics femtosecond pulses demands the further studying. First of all research of the pulses containing small number of the periods is necessary, beginning from 1 [1].

Now time-and-frequency characteristics are considered apart separation from substantive statements of the existing theory of signals, namely, spectra femtosecond impulses are considered as wavelength function. At the same time the modern theory of signals bases on the theory of series and integrals of Fourier, considers spectrum as frequency functions, and there is variety of the fundamental theorems, signals concerning spectral representation, as frequency functions.

At researches femtosecond pulse is considered as a form of an analytical signal. In an analytical signal the information content is concluded in its complex envelope curve. In particular methods which are based on correlation procedures are used. In them energy spectra of signals are Fourier transformations of their autocorrelation functions:

$$G(\omega) = \int_{-\infty}^{\infty} K(\tau)\cdot e^{-i\omega\tau} d\tau$$
$$K(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)\cdot e^{i\omega\tau} d\omega, \quad (6)$$

where G(ω) – power spectrum of signal and K(τ) – autocorrelation function of signal.

Autocorrelation function of an analytical signal is considered as the high-frequency function, envelope curve of which is equal to autocorrelation function of complex envelope curve. But concept of an analytical signal and its complex envelope curve are applied only to narrow-band signals (signal is considered narrow-band, if the quotient $\Delta\omega/\omega_0 \ll 1$ or $\Delta\omega/\omega_0 \le 0.1$, ω_0 – carrier frequency, $\Delta\omega = 2\pi/\tau$, τ – pulse duration). In the modern theory of signals narrow-band it is considered a signal, in the elementary case harmonious oscillation, with number of the periods since $N = 10$ [4].

Theoretical research of spectra femtosecond pulses is spent within the limits of the classical approach which basis is the theory of integrals of Fourier in which bijective relation between time and frequency characteristics of signals is established.

$$S(\omega) = \int_{-\infty}^{\infty} S(t)\cdot e^{-i\omega t} dt$$
$$S(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\cdot e^{i\omega t} d\omega \quad (7)$$

For fuller analysis of femtosecond pulses the line width radiation of the concrete femtosecond laser has been found. As an example has been chosen titanium:sapphire laser for with wave length λ=780 nm and pulse duration $\tau = 2{,}6 \cdot 10^{-15}$s (one period of oscillation).

III. THE SPECTRAL CHARACTERISTICS OF FEMTOSECOND PULSES

Calculations of energy spectra of harmonic oscillations with number of the periods from one to ten have shown that spectral distribution of energy of impulses with small number of the periods has significant levels near to zero frequency. In figure 1 it is presented calculated amplitude (*a*) and energy (*b*) spectra of 1 period of harmonic oscillation.

The possibility of occurrence of such low-frequency spectral components in a spectrum of optical radiation causes serious doubts. It is necessary to consider, that the first mod is raised by the resonator is in MICROWAVE range, and oscillation with smaller frequencies cannot arise by definition of the resonator. The objective relation between time and frequency characteristics of signals at research femtosecond pulses has been called in a doubt [2, 3], that proves by availability of significant levels of energy near zero frequency, at that femtosecond pulses are formed in optical range.

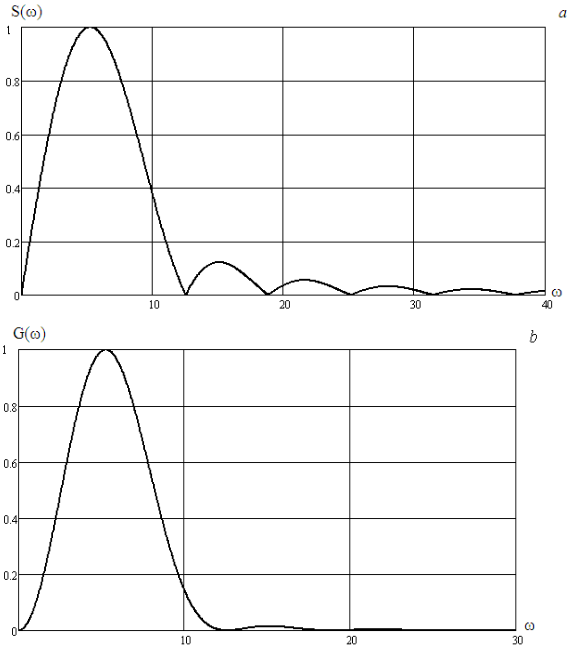

Fig. 1.
a – amplitude spectrum of 1 period of harmonic oscillation,
b – energy spectrum of 1 period of harmonic oscillation

On the other hand on the basis of Wiener-Paley theorem energy of a single pulse signal is continuously distributed in an infinite strip of frequencies, and its value can become zero only on countable number of values which are radicals of this whole function. Thus, presence spectral components near to zero frequency and at the frequencies going to infinity, obviously, are improbable.

As have shown calculations, oscillation with number of periods N=10 is narrow-band. The energy spectrum of this process is presented in figure 2.

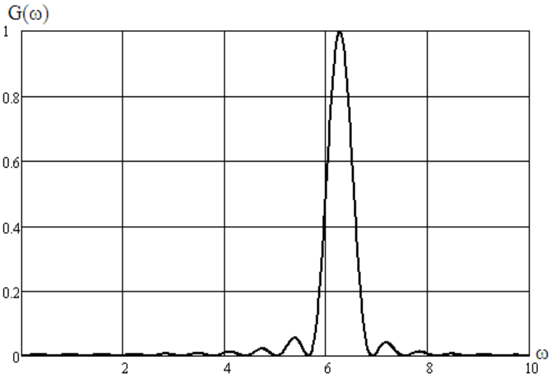

Fig. 2. Energy spectrum of oscillation
with number of periods N=10

Using condition of the narrow-band for a signal containing 10 periods:

$$\frac{\Delta\omega}{\omega_0} = \frac{0.628}{6.274} = 0.1 \qquad (8)$$

The received result can be characterized so: signal with number of the periods 10 and more is narrow-band. Hence, to them methods which are based on correlation methods can be applied. In them energy spectra of signals are Fourier transformations of their autocorrelation functions (6). However, in the modern theory of signals, both theoretical, and experimental methods of researches aren't developed for impulses with small number of the periods (less than 10). Such signals find now the increasing application in various range of application.

Femtosecond pulses are formed in laser systems where appearance of those or other generation rates depends on properties of the active medium and the resonator. The active medium gives spectral broadening which includes homogeneous and inhomogeneous broadenings. At homogeneous broadening each separate atom emission line broaden to the same degree. When resonance frequencies of separate atoms are broaden in some frequency band and the line of all system appears broadened at absence broadenings lines of separate atoms it is a question about inhomogeneous broadening.

There are two mechanisms homogeneous broadening: collisional broadening (in solid it is caused by interaction of atom with phonon lattices) and broadening, caused by spontaneous emission. Inhomogeneous broadening which is connected with movement of atoms is called doppler broadening. There are theoretical methods of calculation homogeneous and inhomogeneous broadenings. With their help they have been defined for the concrete laser with the active medium.

For titanium: sapphire laser (wave length λ=780 nm and pulse duration τ=2,6*10^{-15}s (one period of oscillation)) broadening a spectral line has made $\Delta\nu$=27,03MHz (all mechanisms broadening a spectral line are considered). Doppler broadening has made $\Delta\nu_d$=27MHz. homogeneous broadening, connected with the phenomenon of spontaneous emission $\Delta\nu_{sp}$=0,03MHz, and collisional broadening, had appeared very insignificant (2 kHz). On figure 3 the spectrum of an analized pulse with the designated borders of an optical range and a line width emission is shown. Calculation of energy within the limits of the classical approach has shown that in the given frequency band 1*10^{-6} % of energy of all pulse contain only. Though in this frequency band all mechanisms broadenings a spectral line that involves the maintenance practically all energy of an pulse are considered. Also on classical concepts the signal at which width of the spectrum makes Δv=27,03МГц, has duration 0,23 μs, i.e. much more surpasses duration of femtosecond pulses.

On figure 3 – calculated normalized energy spectrum of length of a sinewave duration 1 period. The radiation maximum is in near infrared area. At the left near IR the area is limited to frequency of 0.13*10^{15} Hz, and on the right 2.45*10^{15} Hz. Also in figure the range of visible light to frequency of 4,71*1015 Hz isn'ted. After visible area of a spectrum begins UV a range of frequencies. Except these ranges in figure 3 it is shown line spreading of the analized laser, it represents a strip black colors round the frequency corresponding to a maximum.

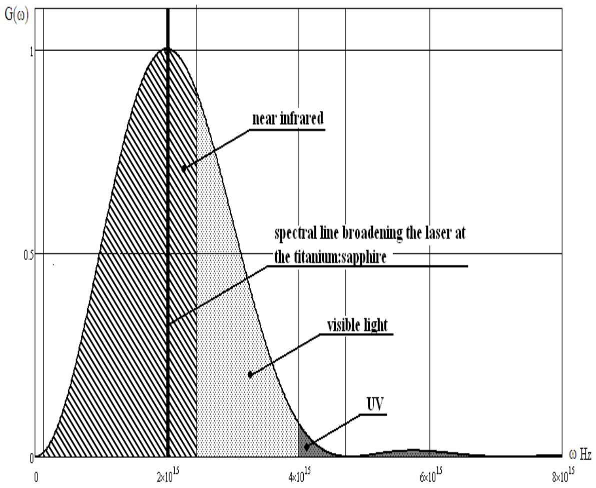

Fig. 3. Energy spectrum of oscillation with λ=780 nm and $\tau=2{,}6*10^{-15}$s

The results of calculations presented in figure 3, demand the further researches in question of an determination of time-and-frequency characteristics femtosecond pulses. That's why the theoretical description of femtosecond pulses demands considering their quantum nature. Elements of the theory of signals in view of their quantum nature are shown in works [6, 7, 8].

IV. THE QUANTUM DESCRIPTION OF FEMTOSECOND PULSES

The femtosecond pulse spectrum in its quantum description is considered, first, from positions of definition of the primary goal of the spectroscopy consisting in photons function distribution establishment on frequencies, secondly, spectra of femtosecond pulses are considered from a position distribution establishment on frequencies that is defined by properties of the active medium.

At the quantum approach it is considered that in spectra of femtosecond pulses, there can be only carriers of energy (photons) with those frequencies for which equal to $h\nu = E_m - E_n$, where ν– frequency of optical radiation , E_m, E_n – power conditions of atom (molecules, ion). Presence of these spectral components, according to the primary goal of spectroscopy, should be considered as a result of spectral measurements.

V. CONCLUSION

The completed calculations within the limits of the classical theory of signals have shown that frequencies near to zero content large part of spectral distribution of pulse energy containing small number of the periods of oscillation. This result isn't coordinated with representation of femtosecond pulse as quantum system, where photons as carriers of energy of electromagnetic field have frequencies in optical range. The given work accent on research of spectral characteristics of oscillations with small number of the periods in the range of low frequencies, in particular in frequencies near zero, however, the classical theory of signals demands presence of carriers of energy on significantly higher frequencies that is not possible. The question of sense of spectral components was discussed with significantly higher frequencies in works [6, 7, 8] and not considered here.

Also in work the actual strip line spreading, considering properties of active medium of the laser is calculated. It occupies very small site of the calculated energy spectrum.

The general conclusion of this work is the statement that theoretical research in range of femtosecond pulses should consider their quantum nature. As the further research of time-and-frequency characteristics femtosecond pulses with small number of the periods, taking into account that their representation in the form of an analytical signal incorrectly is necessary.

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