CHOISE OF SUSPENSION'S ELEMENTS OF AUTOOSCILLATING MICROMECHANICAL GYROSCOPE'S INERTIAL MASSES

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Abstract

There are the results of research on effect of design features of inertial mass's elastic suspension on the characteristics of micromechanical gyroscope, that are operating in self-oscillations' mode. In particular such characteristics are the amplitude of the oscillations along the axis of the excitation and the output axis. It is shown that the location of elastic suspension's elements at the angle of 48° makes it possible to significantly increase the amplitude of the output oscillations and, correspondingly, the sensitivity of the sensor.

Key words: micromechanics, angular rate sensor, gyroscope, autooscillation, self-oscillations, elastic suspension, rigidity.

I. INTRODUCTION

There are next points among the main requirements for an elastic suspension elements of micromechanical gyroscopes' (MMG) inertial mass (IM):

– the absence of cross-linking between the translational and rotational motion of IM;

– the assurance of a certain anisotropy of the suspension's elastic properties for the assigned frequency and mode of natural oscillations of IM;

– the linearity of elastic properties, the smallness of the nonlinear effects in all movements of IM [1].

One of the main reasons to achieve these requirements is the occurrence of technological errors in the manufacture of elastic suspensions.

Elastic suspension elements interlink IM and a frame.

II. COORDINATES SYSTEMS

In general, we assume that the base and the frame (that rigidly connected with the base) are quiet in the absolute coordinates system.

The coordinates system OXYZ is rigidly connected to the base. The system $O^T X^T Y^T Z^T$ is rigidly connected to the IM in such a way that its origin is at the center of mass of IM, as shown in Figure 1.

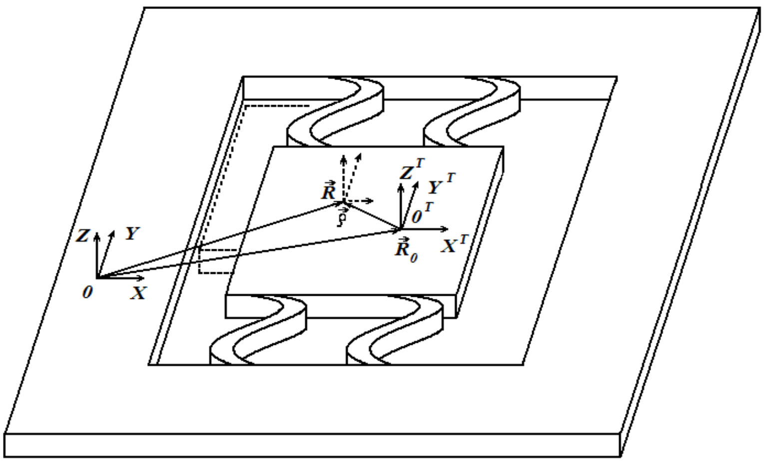

Fig. 1. Arrangement of coordinate system

IM is considered as non-deformable absolutely rigid body that has six degrees of freedom (three translational and three rotational) in the coordinates system OXYZ. Position of IM is determined by the position of system $O^TX^TY^TZ^T$ relatively to the system OXYZ. This position is specified by six independent globalized coordinates: the radius vector and the directional cosines' matrix of dimension 3×3.

The initial (unperturbed) position of IM is characterized by the radius vector $\vec{R}_0$ and matrix K_0. An arbitrary (perturbed) position of IM is characterized by the radius vector $\vec{R}$ and matrix $\boldsymbol{K}$. The transition from the coordinate system $O^TX^TY^TZ^T$ to a system OXYZ is given by displacement vector $\vec{\rho}$ and direction cosines' matrix Θ. The parameters of arbitrary position of MI could be defined by the formulas:

$$\vec{R} = \vec{R}_0 + \vec{\rho}\,;\ K = \boldsymbol{\Theta} K_0 . \qquad (1)$$

III. MATRIX OF RIGIDITY

In general, the dimension of matrix of rigidity is 6×6. This matrix contains the elements c_{ij}. It is symmetrical ($c_{ij} = c_{ji}$). It determines the dependence of the forces and moments from the linear displacement and rotation angles of the IM relatively the equilibrium:

$$[\vec{F}, \vec{M}]^{\mathrm{T}} = \mathbf{C} * [\vec{\rho}, \vec{\theta}]^{\mathrm{T}} . \qquad (2)$$

The matrix of rigidity **C** is

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} . \qquad (3)$$

The upper left quarter of the matrix is the coefficients of the translational rigidity (c_{11}, c_{22}, c_{33}, c_{12}, c_{13}, c_{23}). The lower right quarter of the matrix is the coefficients of the rotational rigidity (c_{44}, c_{55}, c_{66}, c_{45}, c_{46}, c_{56}). The remaining coefficients can be described as crossing or translational-rotational rigidity (c_{14}, c_{15}, c_{16}, c_{24}, c_{25}, c_{26}, c_{34}, c_{35}, c_{36}).

The elastic suspension of IM consists of n elastic elements (Fig. 1 shows a case with n = 4). One built-in support of elastic suspension's element is connected with the frame, and another one is connected with IM.

Let us assume that the elastic elements of the suspension are performed without any technical errors. Then cross-links between translational and rotational motions of IM are absent. In this case, the matrix of rigidity **C** becomes diagonal:

$$\mathbf{C} = \begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} . \qquad (4)$$

Let us assume that the location of the elastic suspension's elements and their form leads to the fact that the coefficient of rigidity c_{33} is much larger than the coefficients c_{11} and c_{22}. As a result IM has only two translational degrees of freedom and one rotational. The hypothesis about these significant differences between the coefficients c_{33} and c_{11}, c_{22} must be verified in the future. Also we make assumption about the complete concurrency of the elastic properties of suspension's elements and about the fact that they are arranged symmetrically to the center of mass of IM. That leads to a lack of the last IM's rotational degree of freedom that was mentioned earlier. In this case, the suspension's matrix of rigidity **C** could be reduced to the dimension 3×3. This matrix determines the dependence of the forces from the linear displacement of the IM relatively the equilibrium:

$$[\vec{F}]^{\mathrm{T}} = \mathbf{C} * [\vec{\rho}]^{\mathrm{T}} . \qquad (5)$$

The matrix of rigidity **C** is

$$\mathbf{C} = \begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{22} & 0 \\ 0 & 0 & c_{33} \end{bmatrix} . \qquad (6)$$

The construction of autooscillating micromechanical gyroscope (AMMG) is described in [2]. The elastic suspension's elements of AMMG should provide linear motion of IM on two axes (OX and OY) and should prevent either the displacement along the axis OZ or its rotational motion. Thus, the requirements for coefficients of matrix of rigidity will be as follows: c_{11} and c_{22} should be much less than c_{33}.

The flexibility matrix of silicon is:

$$S' = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} . \qquad (7)$$

$S_{11} = S_{22} = S_{33} = 0{,}762\ 10^{-11}\ \mathrm{Pa}^{-1}$, $S_{12} = S_{13} = S_{23} = S_{21} = S_{31} = S_{32} = -\ 0{,}214\ 10^{-11}\ \mathrm{Pa}^{-1}$, $S_{44} = S_{55} = S_{66} = 1{,}255\ 10^{-11}\ \mathrm{Pa}^{-1}$, $S_{14} = S_{15} = S_{16} = S_{24} = S_{25} = S_{26} = S_{34} = S_{35} = S_{36} = S_{45} = S_{46} = S_{56} = S_{41} = S_{51} = S_{61} = S_{42} = S_{52} = S_{62} = S_{43} = S_{53} = S_{63} = S_{54} = S_{64} = S_{65} = 0$ [2].

For directions those are determined by the direction cosines l, m, and n, the Young's modulus of silicon is equal to [2]

$$E=\frac{1}{\mathrm{l}^2(\mathrm{l}^2S_{11}+\mathrm{m}^2S_{12}+\mathrm{n}^2S_{13})+\mathrm{m}^2(\mathrm{l}^2S_{21}+\mathrm{m}^2S_{22}+\mathrm{n}^2S_{23})+\mathrm{n}^2(\mathrm{l}^2S_{31}+\mathrm{m}^2S_{32}+\mathrm{n}^2S_{33})+\mathrm{m}^2\mathrm{n}^2S_{44}+\mathrm{n}^2\mathrm{l}^2S_{55}+\mathrm{l}^2\mathrm{m}^2S_{66}}\ . \tag{8}$$

For directions those are determined by the direction cosines l = m = n = 1 the Young's modulus of silicon E = 2,1 10^{10} Pa. The matrix of direction cosines is

$$\begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Then the elastic constant S_{12}' is a component of the elasticity tensor of the quaternary order S_{1122}' and is taking into account the zero elements of the matrix of direction cosines is

$$S_{12}' = S_{1122}' = a_{11}^2\, a_{22}^2\, S_{12} = -\,0{,}214\ 10^{-11}\ \mathrm{Pa}^{-1}. \tag{10}$$

Poisson's ratio is

$$\mu' = -\,S_{12}'\,E = 0{,}0449\ . \tag{11}$$

Modulus of elasticity in shear is [3]

$$\mathrm{G} = \frac{\mathrm{E}}{2(1+\mu')} = 10^{10}\ \mathrm{Pa}\ . \tag{12}$$

IV. RIGIDITY OF RECTANGULAR SRAIGHT ELASTIC SUSPENSION'S ELEMENT

The rectangular straight elastic suspension's element is shown in fig. 2. The rigidity of the k-th such element is defined by the following expressions [3]:

$$c_{1\,(k)} = \mathrm{Ebh}^3\mathrm{l}^{-3};\quad c_{2\,(k)} = \mathrm{Ebhl}^{-1};$$
$$c_{3\,(k)} = \mathrm{Eb}^3\mathrm{hl}^{-3};\quad c_{4\,(k)} = ⅓\mathrm{Eb}^3\mathrm{hl}^{-1}; \tag{13}$$
$$c_{5\,(k)} = \mathrm{G}\beta\mathrm{b}^3\mathrm{hl}^{-1};\ c_{6\,(k)} = ⅓\mathrm{Ebh}^3\mathrm{l}^{-1}.$$

If IM moves linearly relatively to the axis OX IM every k-th elastic suspension' element will be distorted bendingly with the coefficient of rigidity $c_{4(k)}$ and will be distorted extensionally with the coefficient of rigidity $c_{1\,(k)}$.

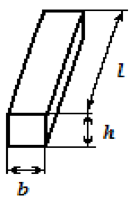

Fig. 2. Rectangular straight elastic suspension's element

Let built-in support of the k-th elastic suspension's element (that is connected with the frame) remains stationary. Let another built-in support of the k-th elastic suspension's element will move along the OX axis to a distance $x_{(k)}$ relatively the equilibrium if this element turns around the OZ axis by an angle θ relatively the equilibrium.

If IM moves along the axis OX then all four elastic suspension's elements will be distorted bendingly and extensionally.

In this case, the elastic bending moment is equal to

$$\mathrm{M}_{\mathrm{ben\ x\ (k)}} = c_{4\,(k)}\,\theta\ . \tag{14}$$

At the same time it is equal to

$$\mathrm{M}_{\mathrm{ben\ x\ (k)}} = \mathrm{F}_{\mathrm{el\ ben\ x\ (k)}}\cdot \mathrm{l}\,, \tag{15}$$

$\mathrm{F}_{\mathrm{el\ ben\ x\ (k)}}$ – elastic bending force of k-th elastic suspension's element that will act on IM if built-in support of this element will move along the OX axis to a distance $x_{(k)}$ relatively the equilibrium.

$\mathrm{F}_{\mathrm{el\ ben\ x\ (k)}}$ could be expressed from (13), (14), and (15):

$$\mathrm{F}_{\mathrm{el\ ben\ x\ (k)}} = \frac{\mathrm{Eb^3hl^{-2}}}{3}\cdot arctg(\frac{\mathrm{x}_{(k)}}{\mathrm{l}})\ . \tag{16}$$

If IM moves along the OX axis then the sum of k projections on the OY axis of elastic stretching forces of elastic suspension's elements will be zero, and the projection on the OX axis will be equal to

$$\mathrm{F}_{\mathrm{el\ str\ x\ (k)}} = \frac{c_{2\,(k)}\cdot\Delta l_{\mathrm{x\ (k)}}\cdot \mathrm{x}_{(k)}}{\sqrt{l^2+\mathrm{x}_{(k)}^{\ 2}}} = \mathrm{Ebhl}^{-1}\cdot(1-\frac{l}{\sqrt{l^2+\mathrm{x}_{(k)}^{\ 2}}})\cdot \mathrm{x}_{(k)}\ , \tag{17}$$

$\Delta\mathrm{l}_{\mathrm{x\ (k)}}$ – increase of length of k-th elastic suspension's element that is caused by the moving of IM along the OX axis to a distance $x_{(k)}$ relatively the equilibrium.

According to the principle of superposition the total elastic force (that will act on IM if it moves along the OX axis) is equal to the sum of the elastic forces of bending and stretching of n elastic suspension's element:

$$\mathrm{F}_{\mathrm{el\ x}} = \sum_{\mathrm{k}=1}^{\mathrm{n}}(\mathrm{F}_{\mathrm{el\ ben\ x\ (k)}}+\mathrm{F}_{\mathrm{el\ str\ x\ (k)}}) = 4Ebhl^{-1}\cdot(\frac{b^2}{3\cdot l}\cdot arctg(\frac{\mathrm{x}_{(k)}}{\mathrm{l}})+ +(1-\frac{l}{\sqrt{l^2+\mathrm{x}_{(k)}^{\ 2}}})\cdot \mathrm{x}_{(k)})\ . \tag{18}$$

The coefficient c_{11} is nonlinear.

If IM moves along the axis OY then all four elastic suspension's elements will be distorted only extensionally. According to the principle of superposition the coefficient c_{22} is equal to the sum of n rigidity coefficients of elastic suspension's elements:

$$c_{22} = \sum_{k=1}^{n} c_{2\,(k)} = 4Ebhl^{-1} . \qquad (19)$$

If $b = 10^{-5}$ m, $h = 2\ 10^{-4}$ m, $l = 10^{-3}$ m then $c_{22} = 168\ 000$ N / m.

Let built-in support of the k-th elastic suspension's element (that is connected with the frame) remains stationary. Let another built-in support of the k-th elastic suspension's element will move along the OZ axis to a distance $z_{(k)}$ relatively the equilibrium if this element turns around the OX axis by an angle ψ relatively the equilibrium.

If IM moves along the axis OZ then all four elastic suspension's elements will be distorted bendingly and extensionally.

In this case, the elastic bending moment is equal to

$$M_{ben\ z\ (k)} = c_{6\ (k)}\ \psi . \qquad (20)$$

At the same time it is equal to

$$M_{ben\ z\ (k)} = F_{el\ ben\ z\ (k)} \cdot l, \qquad (21)$$

$F_{el\ ben\ z\ (k)}$ – elastic bending force of k-th elastic suspension's element that will act on IM if built-in support of this element will move along the OZ axis to a distance $z_{(k)}$ relatively the equilibrium.

$F_{el\ ben\ z\ (k)}$ could be expressed from (13), (20), and (21):

$$F_{el\ ben\ z\ (k)} = \frac{Ebh^3 l^{-2}}{3} \cdot arctg(\frac{z_{(k)}}{l}) . \qquad (22)$$

If IM moves along the OZ axis then the sum of k projections on the OY axis of elastic stretching forces of elastic suspension's elements will be zero, and the projection on the OZ axis will be equal to

$$F_{el\ str\ z\ (k)} = \frac{c_{2\ (k)} \cdot \Delta l_{z\ (k)} \cdot z_{(k)}}{\sqrt{l^2 + z_{(k)}^{\ 2}}} = \\ = Ebhl^{-1} \cdot (1 - \frac{l}{\sqrt{l^2 + z_{(k)}^{\ 2}}}) \cdot z_{(k)} , \qquad (23)$$

$\Delta l_{z\ (k)}$ – increase of length of k-th elastic suspension's element that is caused by the moving of IM along the OZ axis to a distance $z_{(k)}$ relatively the equilibrium.

According to the principle of superposition the total elastic force (that will act on IM if it moves along the OZ axis) is equal to the sum of the elastic forces of bending and stretching of n elastic suspension's element:

$$F_{упр\ z} = \sum_{k=1}^{n} (F_{упр\ изг\ z\ (k)} + F_{упр\ раст\ z\ (k)}) = \\ = 4Ebhl^{-1} \cdot (\frac{h^2}{3 \cdot l} \cdot arctg(\frac{z_{(k)}}{l}) + \\ + (1 - \frac{l}{\sqrt{l^2 + z_{(k)}^{\ 2}}}) \cdot z_{(k)}) \qquad (24)$$

The coefficient c_{33} is nonlinear.

As it could be understood from (5), (18), and (24), c_{33} will substantially exceed c_{11}, if h and l are substantially greater than b.

Thus, let us consider using of rectangular straight elastic suspension's elements. So if coefficients c_{11}, c_{22}, and c_{33} are examined then it could be concluded that IM will have only one degree of freedom. IM will only be able to move translationally along the axis OX. The requirements for the elastic suspension of AMMG were mentioned above. These requirements are not satisfied.

V. RIGIDITY OF CURVILINEAR ELASTIC SUSPENSION'S ELEMENT

Let us consider the use of curvilinear elastic suspension's elements. An example of such elements is shown in Fig. 3.

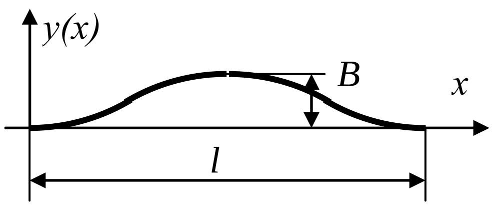

Fig. 3. General view of curvilinear elastic suspension's element

For such form the coefficient of bending rigidity is defined by the following expression:

$$c_{cur\ ben} = (2Ehb^3)(9lB^2)^{-1}. \qquad (25)$$

Let us consider an S-shaped elastic suspension's element that is shown in Figure 4 as a set of two curvilinear elastic suspension's elements. The coefficient of bending rigidity of these elements is the same:

$$c_{cur\ ben\ 1} = c_{cur\ ben\ 2} = c_{cur\ ben}. \qquad (26)$$

In this case, coefficient of bending rigidity of S-shaped elastic suspension's element is defined by the following expression [3]:

$$c_{2\ (k)} = \frac{c_{cur\ ben\ 1} \cdot c_{cur\ ben\ 2}}{c_{cur\ ben\ 1} + c_{cur\ ben\ 2}} = \frac{2Ehb^3}{9lB^2} \qquad (27)$$

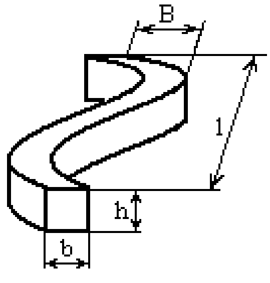

Fig. 4. S-shaped elastic suspension's element

According to the principle of superposition if IM moves along the OY axis then the coefficient of matrix of rigidity c_{22} will be equal to the sum of n coefficients of bending rigidity of S-shaped elastic suspension's elements:

$$c_{22} = \sum_{k=1}^{n} c_{2\,(k)} = \frac{8Ehb^3}{9lB^2} \quad . (28)$$

Thereat if $b = 10^{-5}$ m, $h = 2\ 10^{-4}$ m, $l = 10^{-3}$ m, $B = 5\ 10^{-4}$ m then $c_{22} = 15$ N / m.

If IM moves along the OX axis then the bending strain will have the form that is shown in Fig. 5.

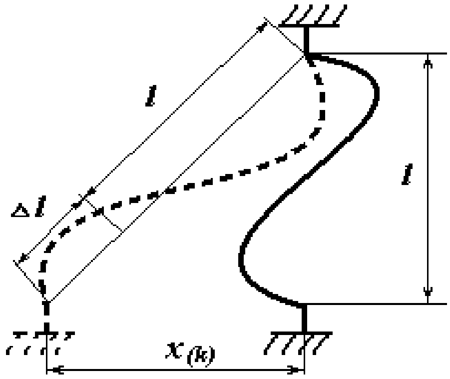

Fig. 5. Bending deformation of S-shaped elastic suspension's element in case of IM's moving along the OX axis

In this case, the elastic bending force $F_{el\,(k)}$ are acting on IM while the built-in support of the k-th elastic suspension's element (that is connected with the IM) is moving along the OX axis to a distance $x_{(k)}$ relatively the equilibrium. This force is equal to:

$$F_{el\,(k)} = c_{2\,(k)} \cdot \Delta l \quad . \quad (29)$$

If IM moves only along the OX axis then sum of the projections of n elastic bending forces $F_{el\,(k)}$ on the OY axis will be zero, and the sum of the projections of n elastic bending forces $F_{el\,(k)}$ on the OX axis will be equal to

$$F_{el\,x\,(k)} = \frac{c_{2\,(k)} \cdot \Delta l_{x\,(k)} \cdot x_{(k)}}{\sqrt{l^2 + x_{(k)}^{\ 2}}} = \frac{2Ehb^3}{9lB^2} \cdot \Big(1 - \frac{l}{\sqrt{l^2 + x_{(k)}^{\ 2}}}\Big) \cdot x_{(k)} \quad , \quad (30)$$

$\Delta l_{x\,(k)}$ – an increase of length of the k-th elastic suspension's element while its built-in support is moving along the OX axis to a distance $x_{(k)}$ relatively the equilibrium.

According to the principle of superposition the total elastic bending force F_{el} of n elastic suspension's elements is:

$$F_{упр\,x} = \sum_{k=1}^{n} F_{упр\,x\,(k)} = \frac{8Ehb^3}{9lB^2} \cdot \Big(1 - \frac{l}{\sqrt{l^2 + x_{(k)}^{\ 2}}}\Big) \cdot x_{(k)} \quad . (31)$$

The coefficient c_{11} is nonlinear.

Similarly the previously considered case of rectangular straight elastic suspension's element the coefficient c_{33} will substantially exceed c_{11}. So it could be suggested that IM haven't got the degree of freedom for linear movement along the axis OZ.

VI. RIGIDITY OF CURVILINEAR ELASTIC SUSPENSION'S ELEMENT THAT IS LOCATED AT AN ANGLE

Let us consider the case of the disposing of curvilinear elastic suspension's elements at the corners of IM at an angle $\alpha = \pi / 4$ relatively to the $O^T X^T$ axis as it is shown in Fig. 6.

The expression of the elastic bending force $F_{el\,(k)}$ is the same as (29). If IM moves along the OX axis then the bending strain will have the form that is shown in Fig. 7.

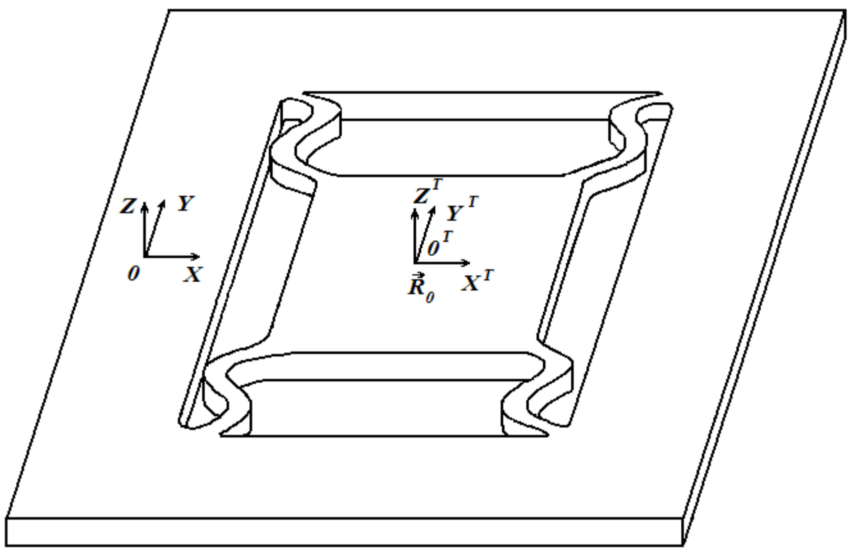

Fig. 6. Case of of the disposing of curvilinear elastic suspension's elements at the corners of IM

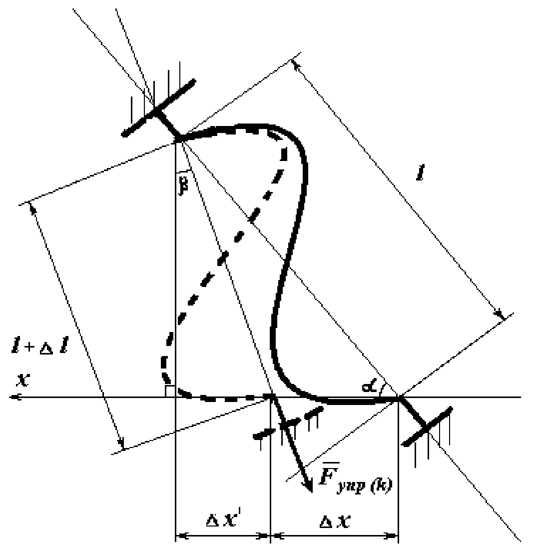

Fig. 7. Bending deformation of S-shaped elastic suspension's element in case of IM's moving along the OX axis, and disposing of this element at the corners of IM at an angle $\boldsymbol{\alpha}$ to the $O^T X^T$ axis

If IM moves along the axis OX to a distance Δx relatively the equilibrium (let us assume that Δx does not exceed the value of $l \cos \alpha$) then the projections of elastic bending forces $F_{el\ x\ (k)\ com}$ of two elastic suspension's elements (toward those IM moves) on the OX axis will be equal to

$$F_{el\ x\ (k)\ com} = -c_{2\ (k)} \cdot \Delta l \cdot \sin \beta =$$

$$= -\frac{c_{2\ (k)} \cdot (\sqrt{\Delta x^2 + l^2 - 2 \cdot \Delta x \cdot l \cdot \cos \alpha} - l) \cdot (l \cdot \cos \alpha - \Delta x)}{\sqrt{\Delta x^2 + l^2 - 2 \cdot \Delta x \cdot l \cdot \cos \alpha}} =$$

$$= -\frac{2Ehb^3}{9lB^2} \cdot (1 - \frac{l}{\sqrt{\Delta x^2 + l^2 - 2 \cdot \Delta x \cdot l \cdot \cos \alpha}}) \cdot (l \cdot \cos \alpha - \Delta x) \quad (32)$$

the projections of elastic bending forces $F_{el\ x\ (k)\ ten}$ of two elastic suspension's elements (from those IM moves) on the OX axis will be equal to

$$F_{упр\ x\ (k)\ ten} =$$

$$= \frac{c_{2\ (k)} \cdot (\sqrt{\Delta x^2 + l^2 + 2 \cdot \Delta x \cdot l \cdot \cos \alpha} - l) \cdot (l \cdot \cos \alpha + \Delta x)}{\sqrt{\Delta x^2 + l^2 + 2 \cdot \Delta x \cdot l \cdot \cos \alpha}} =$$

$$= \frac{2Ehb^3}{9lB^2} \cdot (1 - \frac{l}{\sqrt{\Delta x^2 + l^2 + 2 \cdot \Delta x \cdot l \cdot \cos \alpha}}) \cdot (l \cdot \cos \alpha + \Delta x) \quad (33)$$

According to the principle of superposition the total elastic bending force $F_{el\ x}$ of n elastic suspension's elements (the force that acts along the OX axis) is equal to the sum of $F_{el\ x\ (k)\ com}$ and $F_{el\ x\ (k)\ ten}$:

$$\begin{aligned} F_{el\ x} &= \sum_{k=1}^{2} (F_{el\ x\ (k)\ com} + F_{el\ x\ (k)\ ten}) = \frac{4Ehb^3}{9lB^2} \cdot \\ &\cdot ((1 - \frac{l}{\sqrt{\Delta x^2 + l^2 + 2 \cdot \Delta x \cdot l \cdot \cos \alpha}}) \cdot (l \cdot \cos \alpha + \Delta x) - \\ &- (1 - \frac{l}{\sqrt{\Delta x^2 + l^2 - 2 \cdot \Delta x \cdot l \cdot \cos \alpha}}) \cdot (l \cdot \cos \alpha - \Delta x)) = \\ &= \frac{4Ehb^3}{9lB^2} \cdot (2 \cdot \Delta x + \frac{l \cdot (l \cdot \cos \alpha - \Delta x)}{\sqrt{\Delta x^2 + l^2 - 2 \cdot \Delta x \cdot l \cdot \cos \alpha}}) - \\ &- \frac{l \cdot (l \cdot \cos \alpha + \Delta x)}{\sqrt{\Delta x^2 + l^2 + 2 \cdot \Delta x \cdot l \cdot \cos \alpha}}) \end{aligned} \quad . (34)$$

Similarly the previously considered case according to the principle of superposition the total elastic bending force $F_{el\ y}$ of n elastic suspension's elements (the force that acts along the OY axis) is equal to the sum of $F_{el\ y\ (k)\ com}$ and $F_{el\ y\ (k)\ ten}$:

$$\begin{aligned} F_{упр\ y} &= \frac{4Ehb^3}{9lB^2} \cdot (2 \cdot \Delta y + \\ &+ \frac{l \cdot (l \cdot \sin \alpha - \Delta y)}{\sqrt{\Delta y^2 + l^2 - 2 \cdot \Delta y \cdot l \cdot \sin \alpha}}) - \\ &- \frac{l \cdot (l \cdot \sin \alpha + \Delta y)}{\sqrt{\Delta y^2 + l^2 + 2 \cdot \Delta y \cdot l \cdot \sin \alpha}}) \end{aligned} \quad . (35)$$

The coefficient c_{33} substantially exceed c_{11}. So it could be suggested that IM haven't got the degree of freedom for linear movement along the axis OZ.

Dependence of coefficients $c_{11} = c_{22}$ on the distance of IM's movement along the corresponding axis is presented in Fig.8.

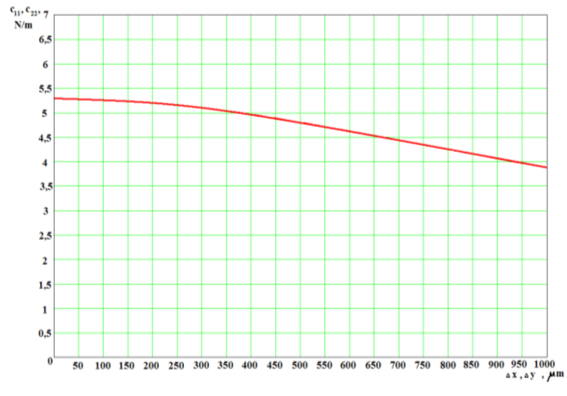

Fig. 8. Dependence of coefficients of matrix of rigidity $c_{11} = c_{22}$ on the distance of IM's movement along the corresponding axis in case of $\boldsymbol{\alpha} = 45^\circ$

VII. MODELLING OF AMMG'S DYNAMICS

The equations of AMMG's dynamics and the model of this dynamics in software environment Simulink is presented in [4]. Let us suppose that S-shaped elastic suspension's elements disposed at the corners of IM at an angle $\alpha = \pi / 4$ relatively to the $O^T X^T$ axis and there is no any energy exchange between the IM through the frame.

Next parameters are used in the modelling:

m = $1.2 \cdot 10^{-6}$ кг (IM);

$\mu_x = \mu_y = 5 \cdot 10^{-5}$ N·s/m (damping coefficients of OX and OY axes);

ω_z = 1°/c (angular rate of AMMG's rotation around OZ axis);

$F_a = 150 \cdot 10^{-6}$ N (force of driver);

$x_m = 10 \cdot 10^{-6}$ m (distance between position sensors (PS));

$U_{п\ d}$ = 5V (input voltage of driver);

$k_d = 15 \cdot 10^{-6}$ N/V (conversion factor of driver);

k_{PS} = 900 V/m (conversion factor of PS);

k_{OC} = 100 (conversion factor of optical converter);

$T_2=10^{-8}$ s (time constant of driver)

There is a dependence of IM's amplitude along driving axis OX on an angle of disposition of elastic suspension's elements α in Fig. 9. There is a dependence of IM's amplitude along output axis OY on an angle of disposition of elastic suspension's elements α in Fig. 10.

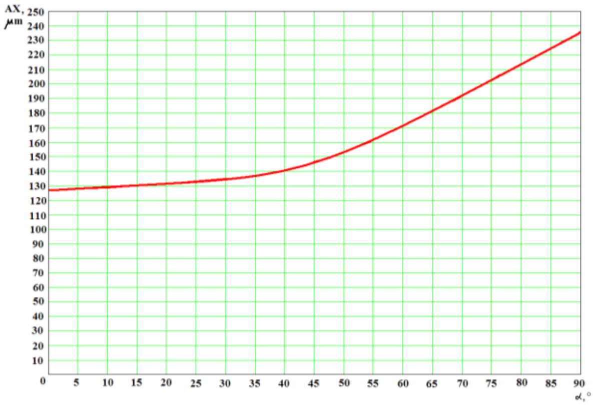

Fig. 9. Dependence of IM's amplitude along driving axis OX on an angle of disposition of elastic suspension's elements α

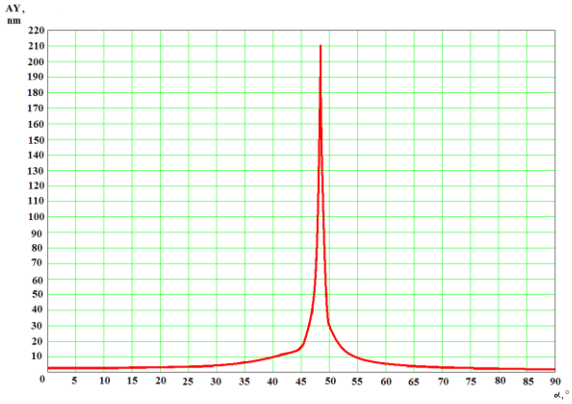

Fig. 10. Dependence of IM's amplitude along output axis OY on an angle of disposition of elastic suspension's elements α

The IM's amplitude along output axis OY will reach its maximum if an angle α = 48˚24'. In this case the recovery time is 0.06 s, the frequency of autooscillations is 273 Hz.

The dependence of the coefficients c_{11} and c_{22} on the distance of IM's movement along the corresponding axis in case of α = 48˚24' is shown in Fig. 11.

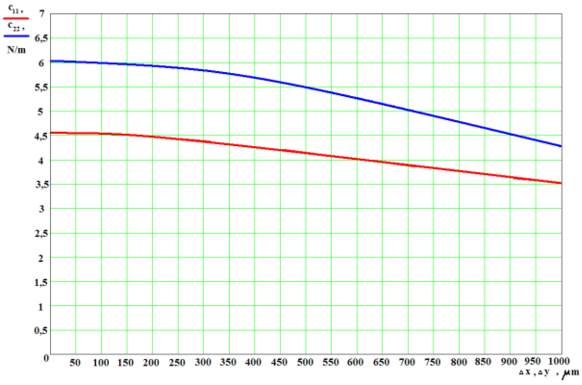

Fig. 11. Dependence of coefficients of matrix of rigidity c_{11} (red one) and c_{22} (blue one) on the distance of IM's movement along the corresponding axis in case of α = 45 ˚

VIII. CONCLUSIONS

Research work has shown that the choice of angle arrangement of elastic suspension's elements makes it possible to optimize the rigidity characteristics of the suspension. And these characteristics significantly influence the parameters and the character of the IM's motion. Particularly important is the result showing the presence of a significant increase in amplitude of the output oscillations (axis OY) at an angle $\alpha \approx 48$ °. The oscillation amplitude 210 nm provides high sensitivity and good dynamic range of measurement.

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