AUTOOSCILLATION INERTIAL MEASURING DEVICES

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Abstract

There're a short constructive description and a physical bases description of a micromechanical angular rate sensor (ARS) with magnetoelectric drivers in this paper.

Advantages of time modulation (TM) and frequency modulation (FM) are shown. Using of compensatory type working devices in a self-oscillations mode is suggested as a method of such types of modulation realization. The results of comparative analysis of different type of drivers are shown. A method of magnetoelectric drivers realization is specified. The kinematic scheme of micromechanical ARS inertia mass (IM) is considered. Also there are a description of micromechanical ARS IM dynamics and a getting of its analytical solution.

Index Terms: micromechanics, autooscillation, angular rate, sensor, electrostatic driver, magnetoelectric driver, time modulation, frequency modulation, excitation, dynamics, nonlinearity, analytical solution.

I. INTRODUCTION

As predicted by the American company «Draper Laboratory» there will be only two types of inertial devices in the future. The niche of high-precision, angular-rate sensors will be taken by fiber-optical gyroscopes and the niche of other inertial measurements will be fully occupied by micromechanical sensors [1]. So, there are many different and important tasks in the micromechanical area at the moment. And these are very potentially important [2].

The gigantism in science and industry, namely attempts to improve performance by increase of size, is over now. Qualitative improvement of designed instruments is the need of today, as well as miniaturization. Therefore research in the area of microelectronic mechanical systems is very important at the moment.

The basic advantages of the micro-mechanical sensors are the small size (such a sensor can be placed on one's nail) and the mass-production that allows for the extremely low prime cost of the sensor. Such sensors are widely used at present in motor car construction, electronics, medicine and communications. The fundamental point of improvement of these micro-mechanical devices is their precision and sensitivity enhancement. When this problem is solved, such systems will be applied in wider spheres. Huge potential of the micro-mechanical devices is recognized by many companies, for instance Draper Laboratory, Analog Devices, British Aerospace etc.

Micromechanical gyroscopes can be used for inertial navigation purposes as a part of a navigation system or stand alone and be used in other applications where rotation rate needs to be measured. Examples of these are automotive applications, such as traction control systems, ride stabilization and rollover detection, some consumer electronic applications, such as stabilization of pictures in digital video camera, inertial mice in computers, and robotics applications. [3]

II. PROBLEM AREA

Engineers often solve the problem of increasing sensitivity in small value and size sensors by reduction of suspension rigidity. As a rule, this reduction leads to the decrease of a measurement range and lowers the accuracy.

The easiest way to solve this problem for engineers is to create the resonant behavior and the amplitude modulation (AM) of a signal. However, studies of the devices constructed according to this principle and also theoretical and experimental research of such modes shows, that significant success in increasing the accuracy of high-sensitivity devices has not been reached.

Reasoning from the theory of information, the self-oscillation regimes that allow passing to FM or TM are of higher potential characteristics than the present-day micro-mechanical devices based on AM [4]. The comparison of received information quantity with various kinds of modulation of an entrance signal is shown on fig. 1 [5]. The autooscillating (or self-oscillating) micromechanical sensors allow for enhancement of both precision and sensibility of a device.

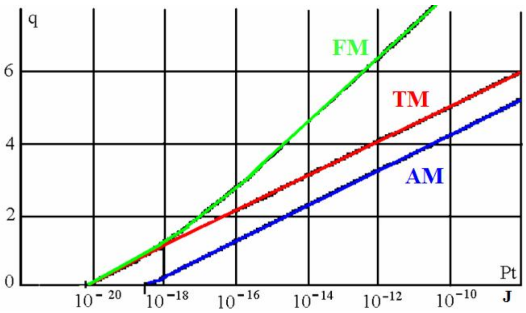

Fig. 1 Information capacity of different types of modulation (q – amount of distinguishable gradations, Pt – input signal power).

III. CHOICE OF DRIVER

Micromechanical inertial sensors are created with electrostatic converters which possess high adaptability to manufacture, but researches have shown that electrostatic elements (position sensors and drivers) of micromechanical sensors possess a number of negative features, which essentially influence the characteristics of devices: small value of reproduced force, nonlinear dependence between force and voltage and nonlinearity of conversion function depending on modulation depth of a capacity backlash. Besides, use of such drivers leads to stability of comparison voltage requirements and electronic part of sensors requirements. Because of small sizes of compensating converters, their capacity is comparable to isolating capacity and capacity of conductors [6].

The potentially important decision allowing, substantially, to solve the problem specified above and allowing to create precision and high-sensitivity, small-sized electromechanical ARS with a digital exit is the combination of high-Q mechanical resonators and the use of the principle of dynamic equilibration using. This principle is realized in devices of compensatory type working in a self-oscillations mode [7].

The kinematic scheme of micromechanical ARS IM is presented in fig. 2, which works in a mode of the self-oscillations, has been offered by the author of this paper. The sensor is carried out with silicon technology with the application of electromagnetic and optoelectronic elements. It is LL-type ARS.

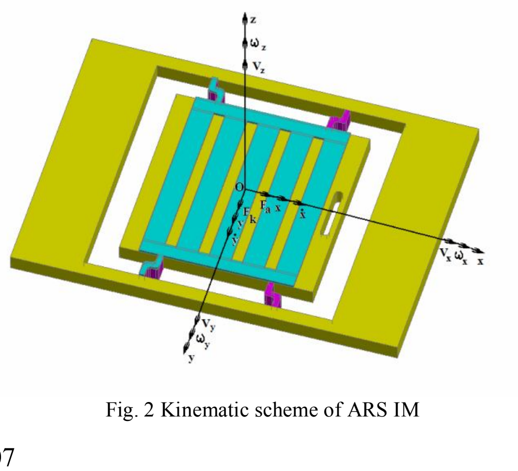

Fig. 2 Kinematic scheme of ARS IM

IT is a monocrystal silicon plate with the rectangular optical gap. This plate is fixed onto the elastic suspension elements. It can make linear moving on two orthogonally related coordinate axes: longitudinal axis OX and lateral axis OY.

Optoelectronic transducers are offered as position sensors (PS). These transducers are constructed on a basis of uncased components. They do not demand application of reference signals. It essentially simplifies the circuitry of the sensor. The first PS fixes movement along the longitudinal axis OX, its optical axis passes through the optical gap. The second PS light is modulated by the edge of IM. The output signal of these PS contains the helpful information about an angular rate.

It is established in the paper [8], that application of a magnetoelectric principle of transformation in the micromechanical drivers allows it to increase its power characteristics approximately by 4000 times in comparison with power characteristics of electrostatic drivers. Therefore, production of such drivers gives the chance (with some complication of the technology) to expand a measurement range and to minimize errors of sensors, and also to realize self-oscillatory modes.

IV. CONSTRUCTION AND PRINCIPLE OF OPERATION

The magnetoelectric drivers are realised as follows in the offered scheme. The conducting paths are dusted on the surface of each IM. These paths are parallel to the lateral axis OY. The length of the paths is l. If electric current are created in the k conducting paths then the magnetic field is created. The induction of this magnetic field is B. Then the operating on IM force $\bar{F}_A$ moves it alongside the longitudinal axis OX. This force is equal to

$$\bar{F}_A = k \cdot l \cdot \bar{I} \times \bar{B}. \qquad (1)$$

This force causes moving of IM along axis OX. In that moment when the IM blocks an optical channel between a light source and a photodetector. The signal from a photodetector leads to change of a direction of a current and the direction of force of Ampere will change on the opposite. Thus, there will be self-oscillations by IM along axis OX.

If micromechanical ARS rotates with angular rate $\bar{\Omega}$ round the sensitivity axis OZ then it leads to occurrence of Coriolis force $\bar{F}_K$. Mass IM is m, the speed of IM along the longitudinal axis OX is $\bar{v}$. This force is equal to

$$\bar{F}_K = -2 \cdot m \cdot \bar{\Omega} \times \bar{v}. \qquad (2)$$

Owing to action of force $\bar{F}_K$ the IM makes secondary self-oscillations along the lateral axis OY, thus the light stream of the second PS is modulated by the edge of the IM. An output signal of these PS contains information about measured angular speed.

V. DINAMICS OF MICROMECHANICAL MAGNETOELECTRIC ARS IM

Let us consider the kinematic scheme of one ARS IM, presented in fig. 3. The coordinate system 0XУZ rigidly bounds with a bearing frame of the sensor in this scheme. The IM in relation to the case and to a bearing frame has two translational degrees of freedom. Movements in these degrees are designated as *x* and *y*. Translational movements are limited by springing elements, whose elastic coefficients are equal to c_x and c_y. The magnetoelectric driver creates the force of Ampere F_a. This driver provides gyroscope excitation. As a result the IM creates plane vibration along an longitudinal axis OX with relative speed $\dot{x}$. The basic reaction of a gyroscope to a case rotation round the sensitivity axis OZ is Coriolis force $\overline{F}_K$ formation along an lateral axis OY. An interaction of speeds ω_z and $\dot{x}$ leads to plane vibration of IM along a lateral axis 0Y. This vibration contains information about measured angular rate ω_z. However we will consider, generally, that the case is also rotating with absolute angular speeds ω_x and ω_y. Also we consider that the case is translated with absolute linear speeds V_x, V_y, V_z. Besides, we consider that the design of springing elements provides IM with only its linear movements to longitudinal and to lateral directions.

To get the ARS dynamics equations we use beta Lagrange's equations [9]:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \left(\frac{\partial T}{\partial q_i}\right) = -\frac{\partial \Phi(q_1 - q_i)}{\partial q_i} - \frac{\partial \Pi(q_1 - q_i)}{\partial q_i} + m_i \,. \quad (3)$$

In the equations (3) it is designated: q_i – generalized coordinates of the system corresponding to *n* degrees of its freedom; *T* – kinetic energy of the system expressed with help of - the generalized coordinates q_i and the generalized speeds $\dot{q}_i$; *Φ* – dissipative function that is defining dispersion of system energy; *Π* – potential energy of system; m_i – other external forces (moments) evidently not dependent on the generalized coordinates and speeds.

For considered gyroscope

$$q_i = x, y;\ \dot{q}_i = \dot{x}, \dot{y};\ m = F_a = F\,. \quad (4)$$

Generally forces *m* can consider both other external revolting and operating influences. We add coordinate system $0_1X_MУ_MZ_M$, connected with IM, then

$$\overline{V_m} = \overline{V} + \dot{\overline{r}} + \overline{\omega} \times \overline{r},$$
$$\overline{V_m} = V_{xm}\overline{i} + V_{ym}\overline{j} + V_{xm}\overline{k}, \quad (5)$$
$$\overline{V} = V_x\overline{i} + V_y\overline{j} + V_z\overline{k}.$$

$\overline{V_m}$ and $\overline{V}$ – vectors of absolute linear speeds of an IM and a bearing frame;

$$\overline{\omega} = \omega_x\overline{i} + \omega_y\overline{j} + \omega_z\overline{k} \quad (6)$$

$\overline{\omega}$ – vector of absolute angular rate of bearing frame rotation of a bearing frame;

$$\overline{r} = x\overline{i} + y\overline{j} \quad (7)$$

$\overline{r}$ – vector of relative linear movement of IM;

Taking into consideration that

$$\overline{\omega} \times \overline{r} = \begin{bmatrix} \overline{i} & \overline{j} & \overline{k} \\ \omega_x & \omega_y & \omega_z \\ x & y & 0 \end{bmatrix} =$$
$$= -\omega_z y\overline{i} + \omega_z x\overline{j} + (\omega_x y + \omega_y x)\overline{k} \quad (8)$$

from the equation (5) it is found that:

$$V_{xm} = V_x + \dot{x} - \omega_z y,$$
$$V_{ym} = V_y + \dot{y} - \omega_z x, \quad (9)$$
$$V_{zm} = V_z + \omega_x y + \omega_y x,$$

Let axes of coordinate system $OX_MУ_MZ_M$ be symmetry axes of the IM disregarding an optical gap. Then its kinetic energy will be equal to:

$$T = \frac{1}{2}m\left[(V_x + \dot{x} - \omega_z y)^2 + (V_y + \dot{y} - \omega_z x)^2 + \right.$$
$$\left. + (V_z + \omega_x y + \omega_y x)^2\right] \quad , \quad (10)$$

m – IM.

Taking into consideration the basic design factors of a gyroscope, we will write down following expressions for its potential energy and its dissipative function:

$$\Pi = \frac{1}{2}c_x x^2 + \frac{1}{2}c_y y^2, \quad (11)$$
$$\Phi = \frac{1}{2}\mu_x \dot{x}^2 + \frac{1}{2}\mu_y \dot{y}^2, \quad (12)$$

μ_x, μ_y – coefficients of viscosity, defining dispersion of sensitive element vibrations energy in coordinates *x* and *y*.

Substituting expressions (10), (11) and (12) into the equations (3) of generalized coordinates *x* and *y*, after calculations we will get the equations of IM movement IT:

$$m\ddot{x} + \mu_x\dot{x} + [c_x - m(\omega_z{}^2 + \omega_y{}^2)]x - 2m\omega_z\dot{y} -$$
$$-m\dot{\omega}_z y + m\omega_x\omega_y y =$$
$$= -m\dot{V}_x + m(V_y\omega_z - V_z\omega_y) + F, \quad (13)$$
$$m\ddot{y} + \mu_y\dot{y} + [c_y - m(\omega_z{}^2 + \omega_x{}^2)]y + 2m\omega_z\dot{x}$$
$$+m\dot{\omega}_z x + m\omega_x\omega_y x =$$
$$= -m\dot{V}_y + m(V_z\omega_x - V_x\omega_z). \quad (14)$$

The equations (13) and (14) define conditions of the dynamic balance of the forces operating along axes OX_M and $OУ_M$. Because of the sensor is intended for measurement of angular rate ω_z, we consider a special case of the equations (13) and (14) when $\omega_x = \omega_y = 0$, $V_x = V_y = V_z = 0$, and angular rate ω_z is constant. Then

$$m\ddot{x} + \mu_x\dot{x} + (c_x - m\omega_z{}^2)x - 2m\omega_z\dot{y} = F, \quad (15)$$
$$m\ddot{y} + \mu_y\dot{y} + (c_y - m\omega_z{}^2)y + 2m\omega_z\dot{x} = 0. \quad (16)$$

This simplified system of equations describes IM dynamic. It is nonlinear because force *F* is some nonlinear function of *x*. The example of possible nonlinear function *F* is presented in fig. 3, where F_m – peak value of the force created by the magnetoelectric driver, x_m – distance from the center of equilibrium to the point of magnetoelectric driver direction switching.

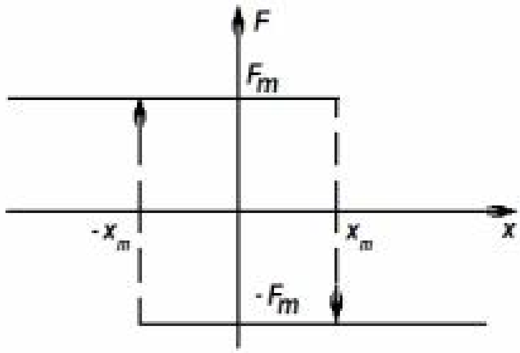

Fig.3. The example of possible nonlinear function *F*.

VI. GETTING OF ANALYTICAL SOLUTION

The simplified model of ARS is based on the system of equations (15) and (16). The dynamic the system and its transients could be examined using this model and programming software Simulink.

Using method of harmonic linearization analytical solution of system of equations (15) and (16) can be gotten.

Then the nonlinear equation (15) should be linearized. Use Fourier transform and separate linear and nonlinear parts then

$$(mp^2 + \mu_x p + c_x - m\omega_z^2)x = F(x). \quad (17)$$

Using method of harmonic linearization we translate this equation into a state space and write it like

$$Q(j\Omega) + R(j\Omega)(q + jq') = 0. \quad (18)$$

$$Q(j\Omega) = -m\Omega^2 + \mu_x \Omega j + c_x - m\omega_z^2,\ R(j\Omega) = 1. \quad (19)$$

According to [10] if nonlinear function has such view as it is shown in Fig.3 then

$$q = \frac{1}{\pi A}\int_0^{2\pi} F(A\sin\Omega)\sin\Omega\, d\Omega = \frac{4F_m}{\pi A}\sqrt{1 - \frac{x_m}{A^2}}, \quad (20)$$

$$q' = \frac{1}{\pi A}\int_0^{2\pi} F(A\sin\Omega)\cos\Omega\, d\Omega = -\frac{4F_m x_m}{\pi A^2}. \quad (21)$$

If substitute equations (20) and (21) into (18) and separate real and imaginary parts then the system of equations will be gotten

$$-m\Omega^2 + c_x - m\omega_z^2 + \frac{4F_m}{\pi A}\sqrt{1 - \frac{x_m}{A^2}} = 0; \quad (22)$$

$$\mu_x \Omega - \frac{4F_m x_m}{\pi A^2} = 0. \quad (23)$$

From (23) we can get that angular frequency of linearization harmonic function is equal to

$$\Omega = \frac{4F_m x_m}{\pi \mu_x A^2}. \quad (24)$$

If substitute equation (24) into (22) then the eighth order equation will be gotten

$$(m^2\omega_z^4 - 2m\omega_z^2 c_x)A^8 - \frac{16F_m}{\pi^2}A^6 + \\ + \frac{16F_m^2 x_m^2}{\pi^2}\left(1 + \frac{2m}{\mu_x}(m\omega_z^2 - c_x)\right)A^4 + \\ + m^2\frac{256F_m^4 x_m^4}{\pi^4 \mu_x^4} = 0. \quad (25)$$

Add new variables

$$a_1 = m^2\omega_z^4 - 2m\omega_z^2 c_x;\ a_2 = -\frac{16F_m}{\pi^2};$$

$$a_3 = \frac{16F_m^2 x_m^2}{\pi^2}\left(1 + \frac{2m}{\mu_x}(m\omega_z^2 - c_x)\right);$$

$$a_4 = m^2\frac{256F_m^4 x_m^4}{\pi^4 \mu_x^4};$$

$$b_1 = \frac{a_2}{a_1};\ b_2 = \frac{a_3}{a_1};\ b_3 = \frac{a_4}{a_1};\ B = A^2;\ D = B + \frac{b_1}{4};$$

$$d_1 = b_2 - \frac{3}{8}b_1^2;\ d_2 = \frac{b_1^3}{8} - \frac{b_1^2 b_2}{2};$$

$$d_3 = \frac{b_1^2 b_2}{16} - \frac{3b_1^4}{256} + b_3; \quad (26)$$

then

$$D^4 + d_1 D^2 + d_2 D + d_3 = 0. \quad (27)$$

To solve this fourth order equitation next third order equitation should be solved

$$F^3 + 2d_1F^2 + (d_1^2 - 4d_3)F - d_2^2 = 0. \quad (28)$$

Add new variables

$$f_1 = 2d_1;\ f_2 = d_1^2 - 4d_3;\ f_3 = -d_2^2;$$

$$G = F - \frac{f_1}{3};$$

$$p = -\frac{f_1^2}{3} + f_2;\ q = \frac{2f_1^3}{27} - \frac{f_1 f_2}{3} + f_3; \quad (29)$$

then

$$G^3 + pG + q = 0. \quad (30)$$

A real root of this equation is

$$G = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2}} + \\ + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2}}. \quad (31)$$

The four roots of the equitation (27) can be found as

$$D_{1,2} = \frac{\sqrt{F} \pm \sqrt{F - 2\left[d_1 + F - \sqrt{(d_1+F)^2 - d_3}\right]}}{2};$$

$$D_{3,4} = \frac{-\sqrt{F} \pm \sqrt{F - 2\left[d_1 + F + \sqrt{(d_1+F)^2 - d_3}\right]}}{2}; \quad (32)$$

Amplitude of linearization harmonic function is equal to

$$A_{1,2} = \sqrt{\frac{\sqrt{F} \pm \sqrt{F - 2\left[d_1 + F - \sqrt{(d_1+F)^2 - d_3}\right]}}{2} + \frac{b_1}{4}};$$

$$A_{3,4} = \sqrt{\frac{-\sqrt{F} \pm \sqrt{F - 2\left[d_1 + F + \sqrt{(d_1+F)^2 - d_3}\right]}}{2} + \frac{b_1}{4}}. \quad (33)$$

So the system of equitation (15) and (16) analytical solution is gotten. This solution now can be used for the further researches.

VIII. CONCLUSIONS

Consequently using of autooscillating inertial measuring devices is a very potentially important area. One of the methods of realization such devices is application of electromagnetic drivers.

Electromagnetic drivers on the basis of Ampere force and with use of a ferromagnetic layer are a new alternative to classical electrostatic drivers. The forces of the electromagnetic driver on the basis of Ampere force are thousands times more than electrostatic forces, and voltage is much less.

The comparative analysis of different types of micromechanical drivers shows that realization a magnetoelectric principles of transformation allows the creation of power characteristics exceeding approximately 4000 times the characteristics of electrostatic drivers. Therefore creation of such drivers allows (in consideration of some complications of the technology) the essential expansion of a measurement range and the minimization of the errors of sensors. [2]

Thus, the application of autooscillating mechanical systems in various measuring devices will allow many improvements of their characteristics. And it is necessary to notice, that described in this paper ARS without introduction of additional elements allows us to measure acceleration. It is also an accelerometer with a sensitivity axis that is parallel to axis OX. The modulation of acceleration measuring signal is either time or frequency.

This and many other areas demand further researches. So it is necessary to define the optimum form of the elastic suspension elements. Also it is necessary to study the possible influence of the accelerations that have not been considered on the device operation. In addition uneven stiffness influence and some other factors should be considered.

The final purpose is the creation of micromechanical ARS with a wider measurement range and bigger accuracy in comparison with the micromechanical devices existing today. Projecting sensors should be close to the fiber-optic ARS in accuracy characteristics. Creation of such a device will allow the expansion of an application area of micromechanical ARS.

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