MATHEMATICAL AND IMITATING MODELS OF ECHO-SIGNALS OF THE SEA SURFACE

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I. INTRODUCTION

One of the most effective methods of research of difficult nonlinear control systems is the method of mathematical modeling of systems on the digital COMPUTER, in particular a method of imitating modeling. At this research, it is necessary to realize on the digital COMPUTER not only algorithm of functioning of the control system, but to synthesize and realize algorithms of imitation of information and interfering input signals of system. The problem of synthesis of mathematical models of input signals appears frequently even more difficult, than realization of algorithm of functioning of investigated system. It, first of all, is connected with the fact, that input signals (both information, and interfering) are casual, and as their mathematical models use casual non-stationary non-Gaussian processes [5, 6].

Modeling casual non-Gaussian processes represents enough difficult task. However, if mathematical models of signals are constructed on the basis of the experimental data, received at supervision of real processes, results of imitating modeling of a control system will reflect features of behavior of real system in real working situations. In this case imitating modeling, as a matter of fact, is the machine experiment, allowing to investigate behavior of system at change of its structure, parameters and change of conditions of its functioning. It, in turn, allows to reduce terms of designing of system and sharply to reduce quantity of natural experiments [3, 5].

In the given work one of possible approaches to synthesis of mathematical models and algorithms of modeling of an input signal of onboard system of processing of the information, caused by reflexions of location signal from a sea surface, is considered. The mathematical model is constructed on the basis of the experimental data, received during natural experiment on analysing of statistical characteristics of echo-signals of a sea surface. The modeled signal can be represented as an information signal of system of data processing (for example, at radiolocation method of definition of force of excitement and a direction of movement of sea waves in control systems of flight of hovercrafts), and a noise(for example in detection systems by search of sea courts).

At construction of mathematical models of input signals of onboard systems in work the method of statistical equivalents is used, thus the mathematical model is understood as the multidimensional joint law of distribution of parameters of the signals, observed on the an output of intake of onboard locator. For synthesis of algorithm of modeling of an echo-signal of a sea surface the method of forming filters, presented in works [2, 4, 9], is used. This method intended for modeling of Gaussian casual processes, is generalized on a modeling case of non-Gaussian casual process [6, 8], in particular – logarithmical normal process. In the given work experimental correlation functions of time fluctuations of an echo-signal and time correlation functions of the modeled processes, which simulates input signals of onboard control systems, are resulted.

II. STATISTICAL MODEL OF ECHO-SIGNALS OF A SEA SURFACE

The signal reflected from a sea surface, represents the narrow-band random process, observed in a strobe of range of an intake of an onboard control system. We will designate duration of a strobe of range through τ_g, usually $\tau_g >> \tau_c$, where τ_c - duration of location of sounding signal. Thus, the echo-signal of a spreading surface, in the specified conditions represents pieces of continuous narrow-band random process by duration τ_g, the following with the period of repetition $T_{РЛС}$. Will consider the most widespread case in practice of non-coherent reception when envelope of observed signal A (t) is used.

Taking into account that the mathematical model should be realized on the digital COMPUTER, it is necessary to replace continuous realizations A (t) with their discrete analogues $\{A_i\}_{i=1,2,\ \ldots,\ M}$ with sufficient for practical use of models by accuracy, where $M = [\tau_g / \Delta T]$ – quantity of count of processes in a strobe of the receiver, taken through a digitization interval ΔT, [·] – function of Antie. To an operating time of all location system are accepted N signal packs, therefore it is observed $N{\cdot}M$ count of envelope. Hence, for definition of mathematical model it is necessary to define$\cdot N{\cdot}M$ - measured joint density (function) of distribution of random variables $\boldsymbol{A_{N\cdot M}} = (A_1, A_2, \ldots, A_{N\cdot M})$.

As one-dimensional density of distribution of probabilities of count of envelope w (A) various density of distribution were used, approximations w (A) in the form of distribution of Releya and in the form of logarithmically-normal distribution are most common. Distribution of Releya is theoretical distribution at representation of a surface of the sea in the form of the big set of separate reflectors, provided that, among reflectors there are no dominating. However at rather small surface of "flare" affect heterogeneity of a sea surface, that does wrongful application of the central limiting theorem. Therefore as w (A) most often use logarithmically-normal model [3, 5–7] as it is acceptable both to the description of fluctuations of envelope and when the density of distribution of Releya is used.

Let's consider double indexation of counts of envelope, count $A_{ij} - j$ count of envelope in i range of strobe, $i=1,2, \ldots, M$, $j=1,2, \ldots, N$. Such double indexation is convenient both by consideration of physics of process, and at construction of algorithms of modeling. The density of distribution A_{ij} can be written down in the form of [5]

$$w(A_{ij}) = \frac{1}{\sqrt{2\pi}\sigma_{ij}A_{ij}}\exp\left\{-\frac{\ln^2\left(A_{ij}/\overline{A}_{ij}\right)}{2\sigma_{ij}^2}\right\}, \qquad (1)$$

where $\overline{A}_{ij}$ and σ_{ij} – the parameters of distribution connected with average value $\widetilde{A}_{ij}$ and dispersion $\widetilde{\sigma}_{ij}^2 = \widetilde{D}_{ij}$ by known expressions, as in [5]. Average value of count $\widetilde{A}_{ij}$ is expressed through average capacity of reflex ions $\widetilde{P}_{ij}$ which is calculated on a basic formula of a radar-location, taking into account concrete parameters of onboard system, conditions of observation and a specific reflecting surface of the sea, and dispersion $\widetilde{D}_{ij}$ is defined through factor of variation $K_{ij} = \widetilde{\sigma}_{ij} / \widetilde{A}_{ij}$ whose experimental numerical values are resulted in many sources, in particular in [5,7]. For small corners of vising this factor with sufficient accuracy for practice can be put equal $K_{ij} = \sqrt{(4-\pi)/\pi} \approx 0{,}52$.

As two-dimensional density of distribution of counts of envelope it is accepted two-dimensional logarithmically-normal density

$$w\left(A_{ij}, A_{nm}\right) = \frac{1}{2\pi\sigma_{ij}\sigma_{nm}A_{ij}A_{nm}\sqrt{1-r_{ijnm}^2}}\exp\left\{-\frac{1}{2\left(1-r_{ijnm}^2\right)}\cdot\right. \qquad (2)$$

$$\left.\cdot\left[\frac{1}{\sigma_{ij}^2}\ln^2\frac{A_{ij}}{\overline{A}_{ij}} + \frac{1}{\sigma_{nm}^2}\ln^2\frac{A_{nm}}{\overline{A}_{nm}} - 2r_{ijnm}\frac{1}{\sigma_{ij}}\ln\frac{A_{ij}}{\overline{A}_{ij}}\cdot\frac{1}{\sigma_{nm}}\ln\frac{A_{nm}}{\overline{A}_{nm}}\right]\right\},$$

where the distribution parameter r_{ijnm} is equal

$$r_{ijnm} = \frac{\ln\left(1 + K_{ij}K_{nm}R_{ijnm}\right)}{\sqrt{\ln\left(1+K_{ij}^2\right)\ln\left(1+K_{nm}^2\right)}}, \qquad (3)$$

and R_{ijnm} – correlation factor between j th count of envelope i th strobe and m-count of envelope n th strobe.

Actually R_{ijnm} describes existential correlation function of echo-signals of a sea surface. Within a strobe time correlation function of count of envelope is practically equal to unit, and basic change R_{ijim} occurs because of spatial diversity of sites of a surface causing these counts of an echo-signal. Therefore it is possible to consider, that R_{ijim} is spatial correlation function of counts of envelope $R^{(\Pi)}(.)$, and R_{ijnj} – time correlation function of count $R^{(B)}(.)$. Taking into account the made remarks existential correlation function in a general view is defined through the sections under formulas

$$\begin{cases} R_{ijnj} = R_{imnm} = R_{in}^{(B)} = R^{(B)}\left(|i-n|T_{\text{p}}\right) = R^{(B)}(\tau), \\ R_{ijim} = R_{njnm} = R_{jm}^{(n)} = R^{(n)}\left(|j-m|\Delta T\cdot c/2\right) = R^{(n)}(\Delta R), \end{cases} \qquad (4)$$

where c – a velocity of light, and ΔR - distance between elements of a sea surface (at small corners of vising in a vertical plane). Sometimes more conveniently instead of ΔR as argument $R^{(\Pi)}$ use time parameter corresponding to distance between j and m counts on a timebase at echo-signal supervision in a range strobe, then $R_{ijim} = R^{(n)}\left(|j-m|\Delta T\right) = R^{(n)}(\tau')$.

At performance of equalities (4) will be carried out approximately and equalities for logarithms of counts of envelopes, i.e. for parameters r_{ijnm}, defined on (3). Equalities for r_{ijnm} we will consider exact

$$\begin{cases} r_{ijnj} = r_{imnm} = r_{in}^{(B)} = r^{(B)}\left(|i-n|T_{\text{p}}\right) = r^{(B)}(\tau), \\ r_{ijim} = r_{njnm} = r_{jm}^{(\Pi)} = r^{(\Pi)}\left(|j-m|\Delta T\right) = r^{(\Pi)}(\tau'), \end{cases} \qquad (5)$$

Then also equalities (4) will be approximately executed. Performance of equalities (5) allows to factorize existential correlation function of logarithms of counts of envelope of an echo-signal r_{ijnm}, namely to present r_{ijnm} in the form of product $r_{ijnm} = r_{ijim}\cdot r_{ijnj} = r_{jm}^{(\Pi)}\cdot r_{in}^{(B)}$. This factorization allows to simplify strongly enough algorithms of modeling of fluctuations of envelope of echo-signals of a sea surface , as under these conditions it is possible to divide the algorithms of modeling realizing spatial fluctuations of echo-signals and time fluctuations.

III. CORRELATIVE-SPECTRAL CHARACTERISTICS OF ECHO-SIGNALS OF A SEA SURFACE

In scientific sources as a rule contains data not about existential characteristics of a signal, and only data and sections of existential correlation function, i.e. about $R^{(\Pi)}(\tau)$ –correlation function inner-period(2) fluctuations, and about $R^{(B)}(\tau)$ – correlation function inter-period fluctuations. Factorization existential correlation function of logarithms of envelope of an echo-signal allows to consider synthesis of algorithms of modeling of spatial and time fluctuations separately. Techniques of synthesis of algorithms are similar, but synthesis of the algorithms realising time fluctuations is more combined, therefore we will consider it.

Let's pass to consideration of time correlation functions $R_{in}^{(B)} = R^{(B)}(|i-n|T_{РЛС})$ – correlation functions inter-period fluctuations of an envelope of an echo-signal of a sea surface. Correlation function of envelope of an echo-signal distributed under the logarithmically-normal law, is connected with correlation function of the logarithm of envelope a parity (fair and for spatial correlation functions) [3, 5, 7]

$$R_{in}^{(B)} = R^{(B)}(|i-n|T_{mo,,}) = \frac{1}{K_i K_n}\left[\exp\left(\sqrt{\ln(1+K_i^2)\ln(1+K_n^2)}\, r^{(B)}(|i-n|T_{РЛС})\right)-1\right], \quad (6)$$

The logarithm of envelope of an echo-signal is distributed under the normal law. Therefore we approximate $r_{in}^{(B)}$ a curve, characteristic for Markovs normal processes

$$r^{(B)}(|i-n|T) = \sum_{p=1}^{N_1} C_p e^{-\alpha_p|\tau|}\cos(\gamma_p\tau) + \sum_{q=N_1+1} C_q e^{-\alpha_q|\tau|}\sin(\gamma_q\tau) + \sum_{l=N_2+1}^{N_3} C_l e^{-\alpha_l|\tau|} \quad (7)$$

where $C_1, C_2, \ldots, C_{N3}, \alpha_1, \alpha_2, \ldots, \alpha_{N3}, \gamma_1, \gamma_2, \ldots, \gamma_{N2}$ – some constants, and $\sum_{p=1}^{N_1} C_p + \sum_{l=N_2+1}^{N_3} C_l = 1$, in particular constants $C_1, C_2, \ldots, C_{N3}, \alpha_1, \alpha_2 \ldots, \alpha_{N3}, \gamma_1, \gamma_2 \ldots, \gamma_{N2}$ it is possible to choose so that logarithmically-normal process of fluctuations of an envelope would be differentiated.

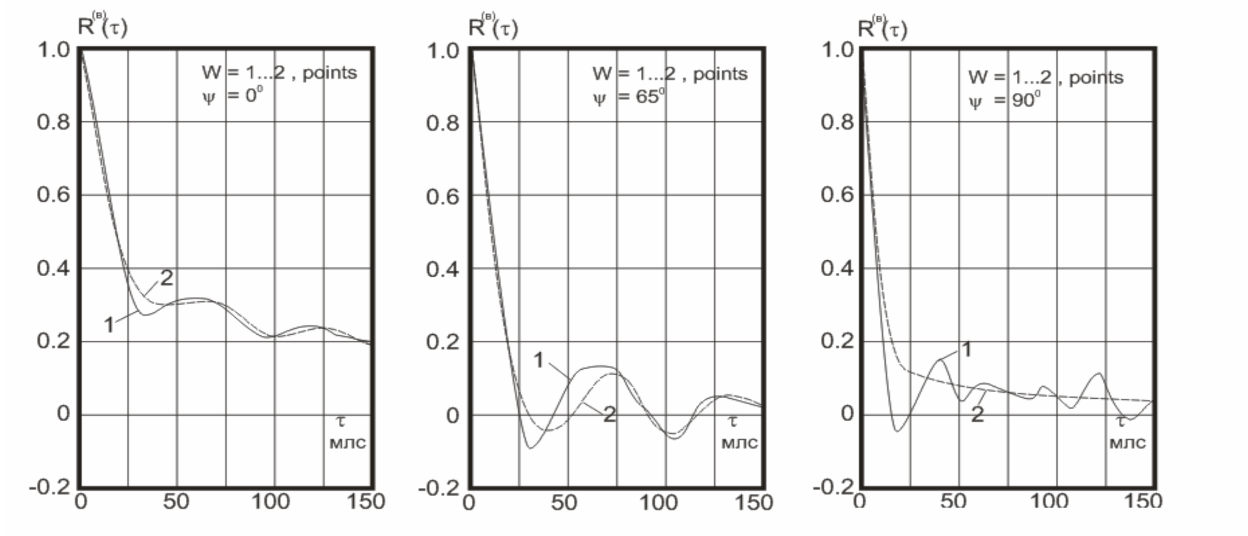
Experimental curves $R^{(B)}(\tau)$ are resulted [5, 7]. Parameters of corresponding approximating curves of a kind (6) and supervision conditions at which experimental time correlation functions have been defined, are shown in tab. 1. On fig. 1, 2 some are resulted experimental curve $R^{(B)}(\tau)$ – the curves marked in fig. 1, and their approximations - the curves marked in fig. 2, at

$$r^{(B)}(\tau) = C_1 e^{-\alpha_1|\tau|}\cos(\gamma\tau) + C_2 e^{-\alpha_2|\tau|} + C_3 e^{-\alpha_3|\tau|}. \quad (8)$$

Table 1.

Parameters of approximating curves and a condition of supervision of experimental time correlation functions

W,point	1…2	2…3	3…5	1…2	1…2	1…2
ψ, deg	60	60	60	0	65	90
α_1, s^{-1}	20	16	5	4	5	51
α_2, s^{-2}	9	14	6	3	50	0,2
α_3, s^{-3}	63	110	252	67	100	184
γ, s^{-1}	110	157	270	103	98	232
C_1	0,11	0,26	0,07	0,04	0,15	0,01
C_2	0,31	0,64	0,20	0,39	0,04	0,07
C_3	0,58	0,10	0,73	0,54	0,81	0,92
$\tau_{0,5}^{(B)}$, s	0,012	0,007	0,004	0,019	0,007	0,005



а) α_1 = 4 s^{-1}, α_2 = 3 s^{-1}, α_3 = 67 s^{-1} γ = 103 rad/s, c_1=0.04, c_2 = 0.39.

б) α_1 = 5 s^{-1}, α_2 = 50 s^{-1}, α_3 = 100 s^{-1} γ = 98 rad/s, c_1=0.015, c_2 = 0.04.

б) α_1 = 51 s^{-1}, α_2 = 0.2 s^{-1}, α_3 = 184 s^{-1} γ = 232 rad/s, c_1=0.01, c_2 = 0.07.

Fig. 1. Time correlation function: 1 - experimental, 2 - model.

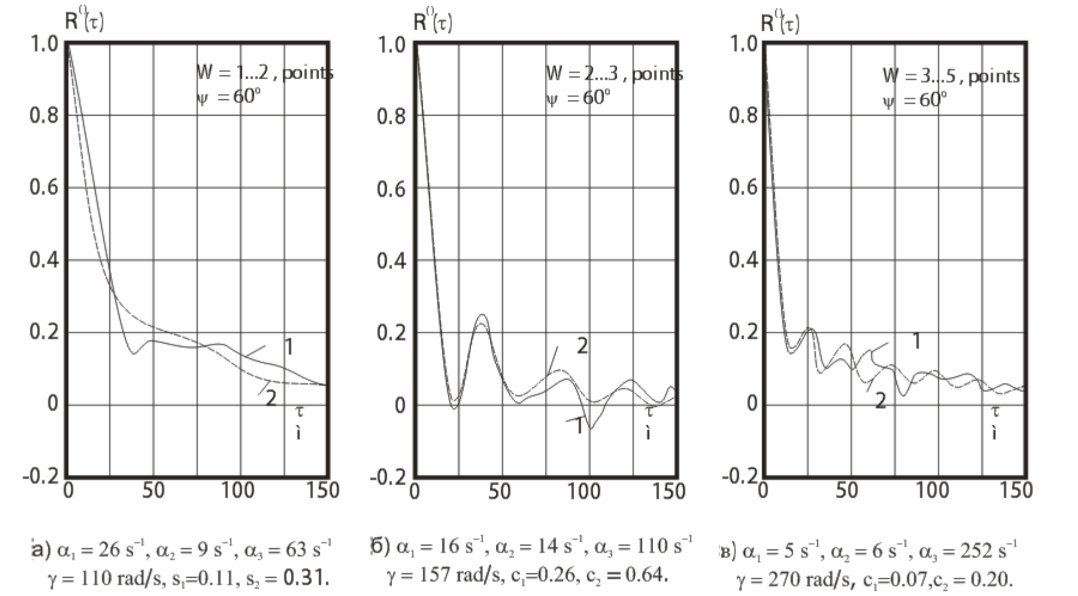


Fig. 2 Time correlation function: 1 - experimental, 2 - model.

The correlation coefficient at approximation of experimental curves was taken by equal 0,52. The analysis of the resulted schedules shows, that for practice approximation of a kind (8) is sufficient, i.e. time fluctuations of envelope of an echo-signal of a sea surface are well enough described by logarithmically-normal Markov process of the fourth order. Spectral characteristics of time fluctuations of echo-signals of a sea surface can be calculated, displaying $R_\tau^{(B)}(\tau)$ on degrees $r^{(B)}(\tau)$ and taking bilateral transformation of Fourier

IV. IMITATING MODELS OF ECHO-SIGNALS OF A SEA SURFACE

Owing to the reasons specified above, we will be limited to construction of imitating models only for modeling of time fluctuations of echo-signals. Further, for simplification of record of expressions we will lower the top index (B). We will notice, that random variables $U_{ij} = \left(\ln A_{ij} - \ln \overline{A}_{ij}\right)/\sigma_{ij}$ are distributed normally with a zero average, an individual dispersion and time correlation function of a kind (7). For modeling of such casual processes it is possible to use the linear discrete forming filters which synthesis is stated in [2,4,9]. Then, as it is easy to see, the algorithm of imitation of fluctuations A_{ij} can be written down in the form of [8]

$$\begin{cases} U_i = \sum\limits_{k=1}^{N} a_i U_{i-k} + \sum\limits_{k=0}^{N-1} b_i \xi_{i-k}, \\ A_i = \overline{A}_i \cdot \exp(\sigma_i \cdot U_i), \end{cases} \qquad (9)$$

In this expression the index j is lowered. Parameters $\mathbf{a} = (a_1, a_2, ..., a_N)$ and $\mathbf{b_N} = (b_0, b_1, \ldots, b_{N-1})$ are defined from system of the equations [4, 7, 8]

$$\begin{cases} \sum\limits_{i=0}^{N-1} r_{i+j} a_{N-i} = r_{N+j}, \quad j = 0,1,\ldots,N-1, \\ \sum\limits_{l=0}^{N-1} b_i b_{i+l} = r_l - \sum\limits_{k=1}^{N} a_k \left(r_{|l-k|} + r_{|l+k|}\right) + \sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N} a_i a_j r_{|l+i-j|}, \\ \qquad l = 0,1,\ldots,N-1, \end{cases} \qquad (10)$$

In which at correlation function r a kind (7) the top index (B) is lowered. For the case considered in given work, N=4, this system can be solved analytically. For $N \geq 5$, the top linear system of the equations concerning vector $\mathbf{a} = (a_1, a_2, ..., a_N)$ has the analytical decision, and the nonlinear system of the equations concerning vector $\mathbf{b_N} = (b_0, b_1, \ldots, b_{N-1})$ dares only numerical methods [1].

V. CONCLUSION

As mathematical model of echo-signals of the sea surface observed in a strobe of range of an onboard control system, it is expedient to use multidimensional logarithmically-normal density of distribution of probabilities of counts of envelope of a signal.

For approximation of time and spatial correlation functions of fluctuations of the logarithm of envelope of an echo-signal it is expedient to use the weighed sum exponential, exponential-cosine and exponential-sinus curves as thus the logarithmically-normal process describing fluctuations of envelope, is Markov process, that simplifies its modeling.

For synthesis of algorithms of imitating modeling it is convenient to use a method of synthesis of forming filters, which allows within the limits of the accepted approximations to synthesize algorithms without methodical errors.

At approximation of correlation functions of time fluctuations of envelope of an echo-signal of a sea surface it is enough to be limited to forming filters of 4th order for which the factors defining the characteristics of filters, can be defined analytically.

The synthesized algorithms of modeling of fluctuations of envelope of an echo-signals of a sea surface are based on a method of statistical equivalents and use experimental data, therefore these algorithms at their practical use for designing and research of difficult control systems actually provide results of machine experiment the most approached to real natural experiment.

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