CALCULATION OF VIBRATING RING GYROSCOPE CHARACTERISTICS

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Abstract

There're a short constructive description and a physical bases description of a vibrating ring gyroscope in this paper.

Methods of all basic parameters calculation are showed. Also there's a description of vibrating ring gyroscope dynamics in a steady-state mode and in transient mode.

The relations between the damping parameter, the duration of transient, the steady-state value of deviation amplitude and the nonlinearity parameter are established on the basis of the modelling.

Index Terms: micromechanics, angular rate, sensor, vibrating ring, gyroscope, solid-state wave, natural frequency, damping parameter, excitation, precession, kinematics, dynamics, nonlinearity, transient, steady-state value.

I. INTRODUCTION

As predicted by the American company «Draper Laboratory» there will be only two types of gyroscopes in the future. The niche of high-precision angular rate sensors will be taken by fibre-optical gyroscopes and the niche of other angular rate measurements will be fully occupied by micromechanical sensors. [1] So there are many different and important tasks in the micromechanical area at the moment. And they're very perspective. [2]

One of the most high-precision micromechanical angular rate sensors is the micromechanical vibrating ring gyroscope created by the «British Aerospace Systems and Equipment Ltd» company. Drift of this sensor makes approximately 5 °/h, that is very good rate for micromechanical devices. [3]

Micromechanical vibrating ring gyroscopes can be used for inertial navigation purposes as a part of a navigation system or stand alone and be used in other applications where rotation rate needs to be measured; examples of these being automotive applications such as traction control systems, ride stabilization and roll-over detection; some consumer electronic applications such as stabilization of pictures in digital video camera and inertial mouse in computers; robotics applications; and, a wide range of military applications such as guidance of missiles and platform stabilization. [4]

II. PROBLEM AREA

Thus, designing and development of a micromechanical vibrating ring gyroscope is the extremely important task. Mathematical models have been constructed; the kinematics, the dynamics, and also the realisations of various behaviours are researched to solve this problem. The project is at a stage of the conclusion of the research and development contract at the moment.

III. CONSTRUCTION AND PRINCIPLE OF OPERATION

The principle of micromechanical solid-state wave gyroscopes operation is based on inert properties of elastic waves. The kinematic scheme of a vibrating ring gyroscope is shown in Fig. 1. A sensitive element of a gyroscope is the elastic thin-walled ring 1. This ring connected with the case 3 of the device by the system of elastic elements. The basic requirement to this suspension system is transfer of the case rotation to the ring at an angular rate of Ω. But also resistance to the radial elastic deformations of a ring should be minimal. Electrostatic drivers 4 are evenly surround the ring 1.

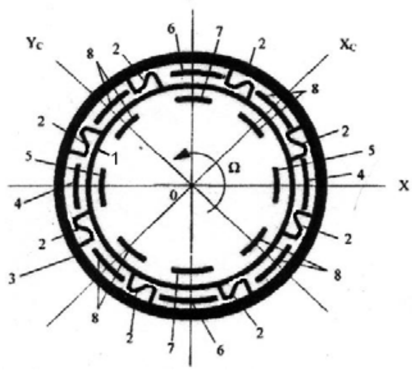

Fig. 1. The kinematic scheme of a vibrating ring gyroscope

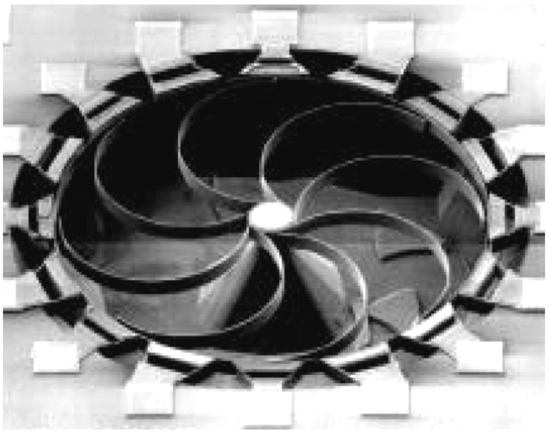

Fig. 2. SEM view of micromechanical vibrating ring gyroscope

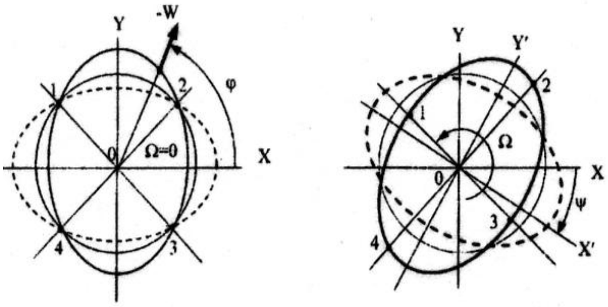

Fig. 3. Vibrations of a wave micromechanical gyroscope when there is no rotation of sensor (left) and when sensor is rotating (right)

SEM view of micromechanical vibrating ring gyroscope is shown in Fig. 2. The ring is 1mm in diameter, 3 μm wide and 35 μm tall. [5]

For an example, external and internal capacitor plates of drivers rigidly connected with the case of the device, and they are located in four zones that coincide with axes of case X and Y in Fig. 1. Existing schemes of excitation systems can differ both in the number of driver zones and in the presence either only external, or only internal driver capacitor plates. And the mobile element of driver is the ring. If some high-frequency voltage will be applied to the electrodes 4 then periodic radial forces will appear. That leads to high-frequency elastic fluctuations of the ring. For micromechanical gyroscopes the basic form of fluctuations is fluctuations with form factor (mode) k = 2. The ring form is close to the elliptic form when the condition is deformed. The form of fluctuations when there's no rotation of the device case (Ω=0) is shown on the left side of the Fig. 3. The radial movings of a ring are close to zero on the points 1, 2, 3 and 4. This points located on the axes that is turned through the angle φ=π/4. If to assume that there is no radial damping forces then the form of the fluctuations that is shown on the left side of the Fig. 3 will exist infinitely long. It will be a standing wave and deviation w from a round shape on the point that is turned through the angle φ will be equal to

$$w = w_0 \cos \nu_c t \cos 2\varphi, \qquad (1)$$

w_0 – maximal deviation, ν_c – natural frequency and excitation frequency, t – time.

The vibrations of the ring can be described by this partial differential equation

$$\ddot{w}'' - \ddot{w} + 4\Omega\dot{w}' + \xi_1(\dot{w}^{VI} + 2\dot{w}^{IV} + \dot{w}'') + \chi^2(w^{VI} + 2w^{IV} + w'') = f(t,\varphi), \qquad (2)$$

$\dot{w} = \frac{dw}{dt}$, $w' = \frac{dw}{d\varphi}$, $f(t, \varphi)$ – φ second derivative of radial forces that is reduced to the dimension of linear acceleration. [6]

Parameters χ^2 and ξ_1 define elastic forces and damping forces. These parameters are equal to

$$\chi^2 = \frac{EJ}{\rho S R^4}, \xi_1 = \frac{\xi J}{\rho S R^4}, \qquad (3)$$

E – coefficient of the ring material elasticity, ρ – density of the ring material, R – radius of the ring centerline, ξ – coefficient of the viscous forces, J – ring moment of inertia. If the ring cross-section is rectangular then

$$J = \frac{Lh^3}{12}, \chi^2 = \frac{Eh^2}{12\rho R^4}, \xi_1 = \frac{\xi h^2}{12\rho R^4}, \qquad (4)$$

L – width of the ring cross-section, h – thickness of the ring cross-section.

IV. CALCULATION OF NATURAL FREQUENCY AND DAMPING PARAMETER

A measured angular speed Ω is a coefficient in the equation (2). Therefore the solution of the equation w(t, φ) will depend on an angular speed Ω if there will be a continuous external excitation $f(t,\varphi)$, or if there will be any own fluctuations of the ring caused by initial conditions.

First of all let's consider the natural vibrations of the ring if there's no any case rotation and there's no external excitation.

If $\Omega(t)=0$, $f(t,\varphi)=0$, $\xi_1=0$ then we'll get from (2)

$$\ddot{w}'' - \ddot{w} + \chi^2(w^{VI} + 2w^{IV} + w'') = 0 \qquad (5)$$

Let's take that initial conditions are t=0, φ=0, $w(0,0)=w_0$, $\dot{w}(0,0)=0$ and partial solution is

$$w(t,\varphi) = w_0 \cos 2\varphi \cos \nu_c t. \qquad (6)$$

Partial derivatives are equal to

$$w'' = -4w_0 \cos 2\varphi \cos \nu_c t,$$
$$w^{IV} = 16 w_0 \cos 2\varphi \cos \nu_c t,$$
$$w^{VI} = 64 w_0 \cos 2\varphi \cos \nu_c t.$$

If we substitute them into the equation (6) then we can calculate the natural frequency of the ring:

$$\nu_c = \frac{6}{\sqrt{5}}\chi = \frac{6}{\sqrt{5}}\frac{h}{R^2}\sqrt{\frac{E}{12\rho}} \ [s^{-1}]. \qquad (7)$$

As an example, if $h=2\cdot10^{-5}$ m, $R=2\cdot10^{-3}$ m, $E=1.87\cdot10^{11}$ N/m^2 (silicon [111]) [7], $\rho=2.3\cdot10^3$ kg/m^3 then from formula (7) we will get ν_c=17.4 kHz.

Next let's consider process of the ring vibration damping. If the case of a gyroscope is deeply vacuumized then damping is basically defined by the energy losses in a body of a ring and in the suspension elements. For this case ($\Omega(t)=0$, $f(t,\varphi)=0$) the equation (2) takes on form:

$$\ddot{w}'' - \ddot{w} + \xi_1(\dot{w}^{VI} + 2\dot{w}^{IV} + \dot{w}'') + \chi^2(w^{VI} + 2w^{IV} + w'') = 0 \qquad (8)$$

Let's find partial solution as

$$w = w_0 e^{-\beta t} \cos \nu_1 t \cos 2\varphi \qquad (9)$$

β – coefficient of natural vibration damping, ν_1 – natural damping frequency.

If we substitute the solution (9) into the equation (8) and separate vibrating forms then we will get the equations

$$-5(\beta^2 - \nu_1^{\,2}) + 36\xi_1\beta - 36\chi^2 = 0$$
$$-10\beta + 36\xi_1 = 0$$

or

$$\beta = \frac{36\xi_1}{10}\ [s^{-1}],$$

$$\nu_1 = \sqrt{\frac{36}{5}\chi^2 - \frac{1}{100}(36\xi_1)^2}\ [s^{-1}] \qquad (10)$$

It's very difficult to calculate coefficient ξ_1 so it's determined experimentally [6]. This determination is based on the degree of damping definition. Let's connect the parameters β and ν_1 with the damping coefficient of the standard oscillatory element δ as $2\delta\nu_1=\beta$.

If we put equations (10) in this formula then we'll get

$$\xi_1 = \sqrt{\frac{80}{36(1+4\delta^2)}}\,\delta\chi,\ \nu_1 = \nu_c\sqrt{1-4\delta^2}$$

or if $4\delta^2 << 1$

$$\xi_1 \cong \sqrt{\frac{80}{36}}\,\delta\chi,\ \nu_1 \cong \nu_c \qquad (11)$$

Parameter δ can be determined experimentally, and then coefficient ξ_1 will be determined too.

As an example, if $\delta=10^{-4}$-10^{-6} m and other ring parameters are determined as it's shown above then ξ_1=8.6-0.086 s^{-1}.

V. CALCULATION OF EXCITATION SYSTEM

Solutions (6) and (9) define forms of natural vibrations of a ring when there's no rotation of a sensor case (Ω(t)=0) and there's no external excitation (*f*(t,φ)=0). If a case will rotate with some angular rate Ω(t) then a standing wave turns through an angle Ψ as it's shown in Fig. 3 (right).

As it's shown in [6, 8] it's necessary to distinguish two basic behaviours of wave solid-state gyroscopes. First behaviour is an inertial mode when *f*(t,φ)=0. Second behaviour is parametrical excitation of a ring [6] when the voltage of driver's excitation system does not depend on angle φ; and angle Ψ is equal to

$$\Psi(t) = \frac{2}{k^2+1}\int \Omega(t)dt \qquad (12)$$

An angle Ψ(t) defines a precession of ring elastic vibrations. Also an angle Ψ(t) bears the information about the angular rate integral. The equation (12) defines integrating behaviour of a wave solid-state gyroscope. Its coordinate Ψ(t) bears the information about angular position of the case.

Micromechanical wave solid-state gyroscopes work in an angular rate sensor mode when case rotation angular rate Ω is constant and the angle Ψ possesses the value [6]:

$$\tan\Psi_0 = \frac{2\Omega}{9\xi_1} \qquad (13)$$

Such behaviour of a gyroscope is provided if there's continuous excitation of a gyroscope when a variable component of excitation *f*(t,φ) (k=2 for the basic form of fluctuations). Then this component's equal to:

$$f(t,\varphi) = f_0\cos\nu t\cos 2\varphi, \qquad (14)$$

f_0 – amplitude of the radial forces specific density that's created by system of excitation of a gyroscope and that's reduced to the dimension of linear acceleration; ν – frequency of excitation. The value of amplitude f_0 depends on the constructive scheme of electrostatic excitation system of a gyroscope.

In the general case

$$f(t,\varphi) = \pm\frac{\varepsilon_0}{2d_0^{\,2}h\rho}[V^2(\varphi,t)]'', \qquad (15)$$

d_0 – capacitive gap between an electrode and a ring; $\varepsilon = 8.85\cdot 10^{-12}\frac{F}{m}$ – an electric constant, a sign depends on external or internal arrangement of electrodes.

As an example, you can see close-up of the capacitive gap in Fig. 4. It's equal to 1.4 µm and tall of the electrode is 60 µm. [4]

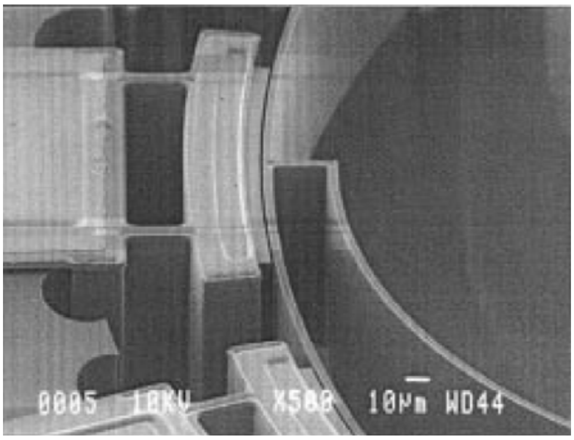

Fig. 4. SEM view of the capacitive gap between the electrode and the ring

If there's a uniform distribution of voltage V(t,φ) and if there's a basic form of ring vibrations (*k*=2) then the voltage's equal to [6]

$$V(t,\varphi) = V_0\cos\frac{\nu}{2}t\cos\varphi, \qquad (16)$$

V_0 – amplitude of an excitation system (with frequency ν/2).

Let's substitute (16) into (15), then:

$$f(t,\varphi) = \frac{\varepsilon_0 V_0^{\,2}}{d_0^{\,2}h\rho}(1+\cos\nu t)\cos 2\varphi\ [ms^{-2}]. \qquad (17)$$

The variable component match for the formula (14) and it's the basic component which is operating on the frequency close to the natural frequency ν_c. The constant component of excitation (17) just leads to an insignificant constant ring tension along an axis OX.

Consistently considering the radial forces created by the separate electrodes, we receive, that if gyroscope excitation simultaneously consist of all internal and external electrodes and if they're located in zones that're turned about the axe OX and the axe OY through the angle $\pm\beta_0$ then

$$f_{4567}(t,\varphi) = \frac{32 e_0 V_0^{2} \sin^2 \beta_0}{\pi^2 a_0^{2} h \rho} \cos \nu t \cos 2\varphi . \qquad (18)$$

VI. EVOLUTION OF RING RESONATOR PRECESSION

If there's rotation of case Ω(t) and there's also excitation of a gyroscope then its evolution is described by the equation (2).

The amplitude of excitation f_0 could be evaluated like (18).

In the general case solution of equation (2) forced component (for the basic form of vibrations k=2) looks like:

$$w(t,\varphi) = (a \cos 2\varphi + b \sin 2\varphi) \cos \nu t + $$
$$+ (m \cos 2\varphi + n \sin 2\varphi) \sin \nu t , \qquad (19)$$

Parameters *a*, *b*, *m*, *n* are time functions. They define evolution of transition from the left to the right form of the ring in Fig. 3. Let us consider steady conditions then parameters *a*, *b*, *m*, *n* will be constant.

Let's calculate partial derivatives of steady conditions and substitute these derivatives in the equation (19). If forms of vibrations will be separated then we'll get this system of the algebraic equations:

$$(5\nu^2 - 36\chi^2) a - 36 \xi_1 \nu m + 8 \Omega \nu n = f_0 ,$$
$$(5\nu^2 - 36\chi^2) b - 8 \Omega \nu m - 36 \xi_1 \nu n = 0 ,$$
$$36 \xi_1 \nu a - 8 \Omega \nu b + (5\nu^2 - 36\chi^2) m = 0 ,$$
$$8 \Omega \nu a + 36 \xi_1 \nu b + (5\nu^2 - 36\chi^2) n = 0 . \qquad (20)$$

Let cavity tuning is optimal. If we solve equation system (20) then we will get

$$a = b = 0 , \quad m = \frac{f_0 36 \xi_1 \nu}{(36 \xi_1 \nu)^2 + (8 \Omega \nu)^2} ,$$
$$n = - \frac{f_0 8 \Omega \nu}{(36 \xi_1 \nu)^2 + (8 \Omega \nu)^2} . \qquad (21)$$

If we substitute these values of parameters *a*, *b*, *m*, *n* into the solution (19) then:

$$w(t,\varphi) = (m \cos 2\varphi + n \sin 2\varphi) \sin \nu t =$$
$$= D \cos(2\varphi + \Psi) \sin \nu t , \qquad (22)$$

where

$$D = \sqrt{m^2 + n^2} = \frac{f_0}{\sqrt{(36 \xi_1 \nu)^2 + (8 \Omega \nu)^2}}$$

is full amplitude of vibrations,

$$\Psi = \arctan \frac{n}{m} = - \arctan \frac{2\Omega}{9 \xi_1}$$

is steady-state rotation angle of an elastic ring vibrations standing wave.

The second special case of the system (20) decision is presence of the natural frequency and resonance frequency detuning, then Δ≠0 and

$$a = \frac{5 f_0 \Delta (2\nu + \Delta)}{[5\Delta(2\nu + \Delta)]^2 + (36 \xi_1 \nu)^2} ,$$
$$b = n = 0 ,$$
$$m = - \frac{36 f_0 \xi_1 \nu}{[5\Delta(2\nu + \Delta)]^2 + (36 \xi_1 \nu)^2} .$$

If we substitute these values of parameters *a*, *b*, *m*, *n* into the solution (19) then:

$$w(t,\varphi) = (a \cos \nu t + m \sin \nu t) \cos 2\varphi =$$
$$= D_1 \sin(\nu t + \Psi_1) \cos 2\varphi , \qquad (23)$$

where

$$D_1 = \sqrt{a^2 + m^2} = \frac{f_0}{\sqrt{[5\Delta(2\nu + \Delta)]^2 + (36 \xi_1 \nu)^2}}$$

is full amplitude of vibrations,

$$\Psi_1 = \arctan \frac{a}{m} = - \arctan \frac{5\Delta(2\nu + \Delta)}{36 \xi_1 \nu}$$

is steady-state rotation angle of an elastic ring vibrations standing wave.

Basing upon (23), we can see that if a frequency detuning will grow then full amplitude of ring vibrations decreases. And phase displacement of these vibrations increases by Ψ_1. It's important to consider this fact when output pickup of the vibrations will be processed.

Output vibrations attenuation because of this detuning Δ can be estimated by parameter k_1

$$k_1 = \frac{D(\Omega = 0)}{D_1} . \qquad (24)$$

Using formulas (22), (23), (24) we can get

$$k_1 = \frac{\sqrt{[5\Delta(2\nu + \Delta)]^2 + (36 \xi_1 \nu)^2}}{36 \xi_1 \nu} .$$

As an example, if Δ=10 Hz and other ring parameters are determined as it's shown above then k_1=203. If Δ=1 Hz then k_1=20.3. If Δ=1 Hz and ξ_1=0.86 then k_1=2.25.

Thus, these calculations show that if the value of ξ_1 is small (natural vibrations of a ring have a big Q-factor) then it's extremely important to provide a resonant operating mode of a gyroscope with high accuracy. It can be provided by realisation of an autogenerating excitation of a ring resonator.

The solutions (22) and (23) of the equation (19) correspond to the steady-state ring vibrations. It was supposed that parameters *a*, *b*, *m*, *n* are constants in the formula (19). Better description of gyroscope dynamics and better description of vibrations evolution of a ring can be obtained if the parameters *a(t)*, *b(t)*, *m(t)*, *n(t)* in the equation (2) will be supposed as a time-functions. In this case substituting the particular solution (19) into the equation (2) and dividing vibration forms, we will get the following system of the differential equations:

$$\frac{d\dot{a}}{dt} = \frac{1}{5} [(5\nu^2 - 36\chi^2) a - 36 \xi_1 \dot{a} - 8 \Omega \dot{b} -$$
$$- 36 \xi_1 \nu m - 10 \nu \dot{m} + 8 \Omega \nu n - f_0] ,$$

$$\frac{d\dot{b}}{dt} = \frac{1}{5} [- 8 \Omega \dot{a} + (5\nu^2 - 36\chi^2) b - 36 \xi_1 \dot{b} - ,$$
$$- 8 \Omega \nu m - 36 \xi_1 \nu n - 10 \nu \dot{n}] ,$$

$$\frac{d\dot{m}}{dt} = \frac{1}{5} [36 \xi_1 \nu a + 10 \nu \dot{a} - 8 \Omega \nu b +$$
$$+ (5\nu^2 - 36\chi^2) m - 36 \xi_1 \dot{m} + 8 \Omega \dot{n}] ,$$

$$\frac{d\dot{n}}{dt} = \frac{1}{5} [8 \Omega \nu a + 36 \xi_1 \nu b + 10 \nu \dot{b} -$$
$$- 8 \Omega \dot{m} + (5\nu^2 - 36\chi^2) n - 36 \xi_1 \dot{n}] . \qquad (25)$$

The equations (19), (25) give the full description of micromechanical solid-state wave gyroscope dynamics with the item excitation that's working in angular speed sensor mode and with positioning excitation. Because of complexity of system (25) author of this paper recommend to research dynamics using a method of numerical integration.

VII. RESULTS OF MODELLING

The model of vibrating ring gyroscope was based on the equations which are described above. System transient was considerer by the instrumentality of this model using programming software Simulink.

Here's the list of parameters that is used for the modeling:

$$h = 2 \cdot 10^{-5}\ m,\ \ R = 2 \cdot 10^{-2}\ m,$$

$$E = 1.87 \cdot 10^{11}\ \frac{N}{m^2},\ \ \Omega = 22\ °/s,$$

$$\xi_0 = 8.85 \cdot 10^{-12}\ °/s,\ \ \beta_0 = 6\ °/s,$$

$$V_0 = 1.6\ V,\ \ d_0 = 10^{-5}\ m.$$

Parameters *m (t)* and *n (t)* transients are shown in fig. 5 and fig. 6. Parameter *m (t)* defines change amplitude of ring vibrations in time at a zone of excitation electrodes, and *n (t)* defines change amplitude of ring vibrations in time at a zone of capacitor position detector electrodes.

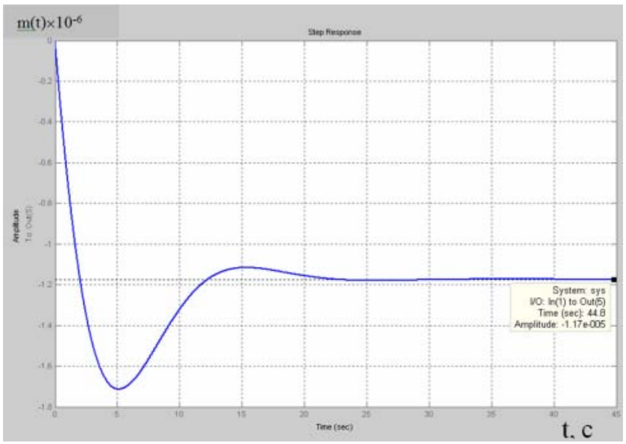

Fig. 5. *m* (t) transient

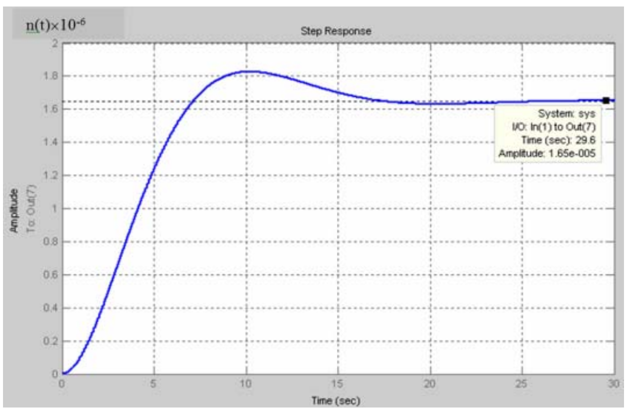

Fig. 6. *n* (t) transient

Evolution of the elliptic vibrations form for several moments of time is shown in fig. 7. Real radial displacements of the elliptic vibrations are increased in 10 times for obviousness.

If parameter ξ =0.060696 (it corresponds to the high Q-factor D=5·10^5), then an output signal of a gyroscope is in a zone of the high nonlinearity. A nonlinearity parameter is equal to

$$\Delta_L = 1 - \frac{(36\xi_1)^2}{(36\xi_1)^2 + 8\Omega^2} = 0.65, \qquad (26)$$

Let's notice that a reduction of parameter ξ_1 quickly enough moves measurement to a linear zone. So, if ξ_1=0.60696 then $\Delta_л$=0.019 (1.9 %), and if ξ_1=0.0696 then $\Delta_л$=0.00019 (0.019 %). The ξ_1-Δ_L relation is shown in fig. 8.

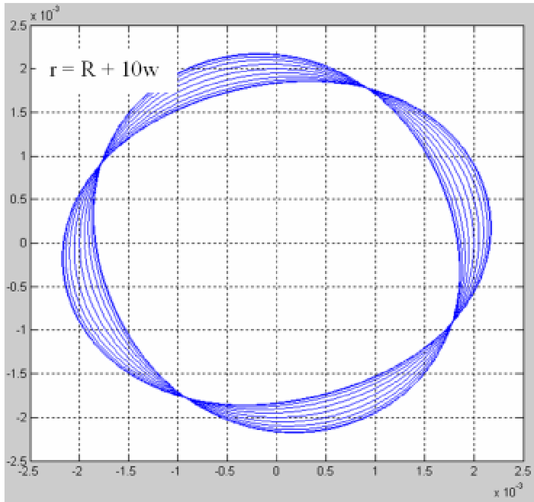

Fig. 7. Evolution of the elliptic vibrations form

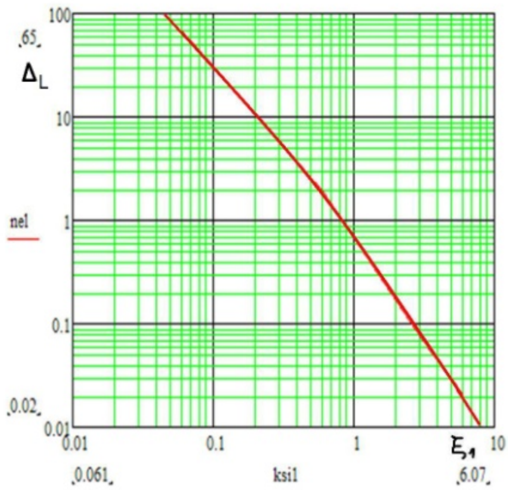

Fig. 8 ξ_1-Δ_L relation

Besides, a small value of the parameter ξ_1 leads to the big duration of *n(t)* transient which makes 30 s according to fig. 6. However, if the parameter ξ_1 is small then the steady-state value of amplitude *n* of ring vibrations at a zone of capacitor position detector electrodes. It's rather big and makes $n = 1.65 \cdot 10^{-6}$ m (16498 Å). ξ_1 – steady-state value of amplitude relation is shown in fig. 9. Also a small value of ξ_1 leads to essential increase of gyroscope time of availability for measurements. ξ_1 – duration of transient relation is shown in fig. 10.

So, the results of modelling show that the essential increase in parameter ξ_1 is required for exception of measurements nonlinearity and for reduction of transients' duration.

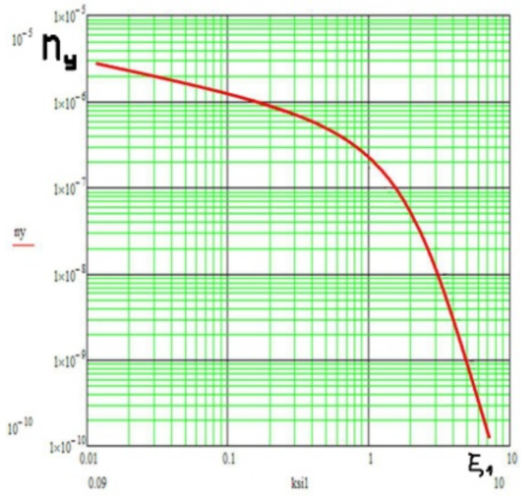

Fig. 9. ξ_1 - steady-state value of amplitude relation

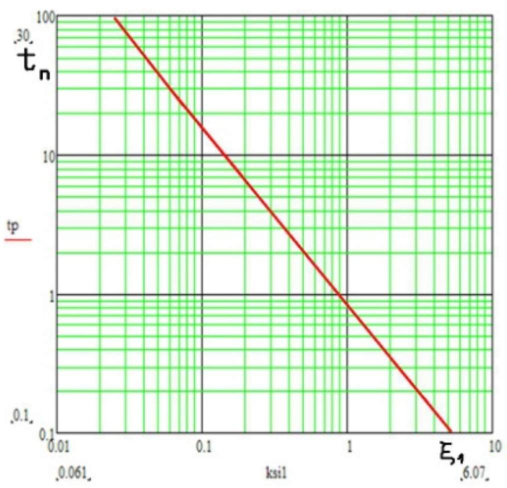

Fig. 10. ξ_1 - duration of transient relation

If parameter ξ_1 will be multiplied 50 or 100 times then the maximum deviation from linearity will be no more than $\Delta_л \cong 0.02$ % (26). Thus duration of transients in comparison with an initial case is reduced 150–300 times.

However, the increase in parameter ξ_1 leads to reduction of radial movings vibration amplitude at a zone of capacitor position detector electrodes. In the cases from fig. 5 and fig. 6 these amplitudes make 19.6 Å and 4.91 Å. At this point let us notice that so small movings can be fixed by modern capacitor position detectors which resolution makes 0.2 Å. However, in this case there is no necessity to work with so small fluctuations of the resonator. In an initial case intensity of the exciting radial forces f_0 was calculated using excitation voltage V_0=l.6 V. Such value of voltage V_0 provided amplitude of radial vibrations of resonator $n = 1.65 \cdot 10^{-6}$ m if half of capacitive gap size equal $5 \cdot 10^{-6}$ m. If parameter ξ_1 will be essentially increased then exciting voltage amplitude can be essentially increased too. If V_0=l6 V then amplitude in fig. 5 and in fig. 6 will be 100 times more. It's possible to make voltage V_0 even more. But however, it should not exceed breakdown voltage that's within 15–25 V/μm. It depends on gyroscope pumpdown degree.

VIII. CONCLUSIONS

In this paper the calculation of vibrating ring gyroscope characteristics was discussed. Thanks to this calculation there is enough information to get performance specification and to begin development of a micromechanical sensor at the moment. The experimental results obtained in a simulated environment have confirmed the benefits of the micromechanical vibrating ring gyroscope approach proposed for the increasing of the angular rate measurement precision. A research and development contract is going to be concluded at the moment.

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