METHOD OF SYNTHESIS OF THE DISCRETE LINEAR FORMING FILTER ANY ORDER

George Kuyumchev

Saint-Petersburg State University of Aerospace Instrumentation,
Saint-Petersburg, Russia

I. INTRODUCTION

The most comprehensible method, allowing to model processes in real time and without a methodical error, is the method of nonlinear functional transformation of normal random process. Mathematical expressions of algorithms of modelling, the following from it, allow to parallelize calculations at their realization on multiprocessing COMPUTERS.

Advantages of this method especially began to be shown at occurrence of modern multiprocessing platforms and corresponding multitask and multithreaded operational systems of type Windows'95, Windows'NT (but not Windows'3.1) [2]. The basic disadvantage of a method is great size of the preparatory work, connected with calculation of parameters of algorithms. Thus, it is required to make decision in process of performance of numerical calculations for separate private problems that does practically impossible automation of process of synthesis, algorithm of modelling.

In this work some new results on synthesis of linear discrete filters of any order, which allow to receive the closed analytical expressions for calculation of coefficients of filters and by that, completely to automate the most labour-consuming part of a technique of synthesis of a method of nonlinear functional transformation, are presented. The presented algorithms are steady, focused on statistical problems, their synthesis was made taking into account the subsequent realization on the COMPUTER that has caused their high efficiency in comparison with known algorithms.

II. MATHEMATICAL MODELS OF RESULTED VECTOR NON-STATIONARY PROCESSES AND ALGORITHMS OF THEIR FORMATION

In the given work the method of synthesis of discrete forming filters (DFF), now is used for imitation of normal processes with set correlative-spectral characteristics, is developed. The received algorithms are mashinno-focused and represent the general model of autoregress-sliding of an average.

Synthesized DFF contain variable coefficient in time, i.e. allow to model non-stationary random processes, but only such non-stationary processes, which for Gauss processes normalized correlation function depends on a difference of the arguments and average value and a dispersion are any functions of time. The given class of non-stationary random processes has received the name of resulted non-stationary processes.

For correlation function of resulted non-stationary processes $\varsigma(t)$

$$K_\varsigma(t_1,t_2) = m_\varsigma(t_1)m_\varsigma(t_2) + \sigma_\varsigma(t_1)\sigma_\varsigma(t_2) \quad r_\xi(|t_1 - t_2|), \quad (1)$$

where $m_\varsigma(t)$ and $\sigma_\varsigma(t)$ – mathematical expectation and mean square deviation of process $\varsigma(t)$ accordingly, the known determined functions of time, at modelling these functions rely to be known. We will define normalized random process generated by process $\varsigma(t)$, in a kind

$$\xi(t) = \frac{\varsigma(t) - m_\varsigma(t)}{\sigma_\varsigma(t)}, \quad (2)$$

then $m_\xi(t) = 0$ and $\sigma_\xi(t) = 1$, and correlation function of process $\xi(t)$ is equal $r_\xi(|t_1 - t_2|) = r_\xi(\tau)$, i.e. process $\xi(t)$ is stationary in a broad sense. Therefore modelling of non-stationary resulted process $\varsigma(t) = m(t) + \sigma(t)\xi(t)$ is reduced to modelling stationary in a broad sense process $\xi(t)$ which algorithm of formation is considered below.

Algorithms of modelling vector Gauss process with any correlative-spectral characteristics are in detail considered in the scientific literature. However, complexity of these algorithms increases as far as increase in the next settlement value, therefore these algorithms are expedient for using for formation small samples. On the other hand it is known, that for modelling Markov processes it is possible to use forming filters with final number of the coefficient, considering the previous values of formed casual process.

For approximations of correlation functions of information signals and the stirring influences meeting in practice, in particular at the description of mathematical models of entrance signals of onboard control systems, it is enough to use correlation functions of Markov random processes not above 4th order. In the majority of cases in general researchers are limited Markov sequences not above the second order and it is caused not only aspiration to raise efficiency of algorithm of modelling, but also complexity of analytical methods of their synthesis. However, at theoretical researches, for example in other subject domains, algorithms of any order can be necessary, therefore the method of synthesis of DFF of any N th order is considered below, and the subsequent this method is generalised on multichannel DFF (MDFF).

Complexity of synthesis of DFF for modelling of fluctuations of input signals of onboard systems is generally caused by four major factors: multichannel, non-Gaussian processes, non-stationarity and discreteness. Last factor concerns the machine-oriented algorithms. There is and simplifying factors: similarity of correlation functions in each of channels of the multichannel filter, symmetry of channels on normalized characteristics and mutual correlation functions, and also stationarity of normalized processes, which simulate fluctuations of parametres of reflected signals, i.e. modelled processes are resulted.

Method of synthesis of non-stationary nonlinear multichannel DFF (NMDFF) we will divide on two parts: synthesis of channel linear DFF (LDFF) for modelling of the normal stationary process, considered in this subsection, and synthesis of the interchannel matrix filter, providing set interchannel correlation dependence, considered in following subsection.

III. METHOD OF SYNTHESIS OF THE LINEAR DISCRETE FORMING FILTERS OF ANY ORDER

Let's consider discrete transfer function of recursive DFF N th order, which in a general view can be written down as

$$H(z)=\frac{\sum_{i=0}^{N\text{-}1} b_i Z^{-i}}{1\text{-}\sum_{j=1}^{N} a_i Z^{-j}}=\frac{U_k}{\xi_k}, \qquad (3)$$

where $\mathbf{a}=(a_1,a_2,\ldots,a_N)$ and $\mathbf{b}=(b_1,b_2,\ldots,b_N)$ – vectors of parametres DFF, U_k – its output signal, and ξ_k – discrete white normal noise with a zero average and an single dispersion, $\xi_k \sim N(0,1)$, Z^{-1} – a delay on one step. From here output signal U_k is equal

$$U_k=\sum_{j=1}^{N} a_j Z^{-j} U_k+\sum_{i=0}^{N\text{-}1} b_i Z^{-i}\xi_k=\sum_{j=1}^{N} a_j U_{k-j}+g_k, \qquad (4)$$

where g_k – the painted noise

$$g_k=\sum_{i=1}^{N-1} b_i \xi_{k-i}\,. \qquad (5)$$

There are many different ways of synthesis of DFF on the set correlative-spectral characteristics of output signal. The main from these ways is synthesis of DFF on spectral density of a output signal by it factorization, however this method in practice can be applied successfully only for filters of the second order (theoretically for filters of 4th order) owing to the difficulties, arising at factorization of spectral density as function of pseudo-frequency. The methods of synthesis demanding the task of correlation function in several points are developed, however these methods, first, lead to DFF of very high order, and secondly, they do not guarantee the behaviour of correlation function of modelled process out of an interval of the task of its correlation function. Here is offered other method of synthesis of DFF, materially, being intermediate between a statistical method and the methods of synthesis following from the theory of automatic control.

Let lattice function of output signal of DFF equal $r_{|i-n|}$ corresponds to Markov`s process of N-th order. It means, that next counted value U_k should depend only from N the previous values, that is can be presented in the form of (4), and the algorithm can be realised in the form of DFF with transfer function of a kind (3). It is easy to notice, that average value $\tilde{U}_k=M[U_k]=0$ at any k, that is at any k U_k can be pre sented in the form of the sum of independent random variables ξ_k, $j<k$ with distribution $\sim N(0,1)$. Or, as $\tilde{U}_k=M[U_k]=M[U_m]=\tilde{U}_m$,

$$\begin{aligned}\tilde{U}_k&=M\left[\sum_{j=1}^{N} a_j U_{|k-j|}+\sum_{i=0}^{N-1} b_i \xi_{|k-i|}\right]=\\&=\sum_{j=1}^{N} a_j M\left[U_{|k-j|}\right]+\sum_{i=0}^{N-1} b_i M\left[\xi_{|k-i|}\right]=\tilde{U}_j\sum_{j=1}^{N} a_j.\end{aligned} \qquad (6)$$

$$\tilde{U}\cdot\left(1-\sum_{j=1}^{N} a_j\right)=0, \qquad (7)$$

But for $\left(1-\sum_{j=1}^{N} a_j\right)\neq 0$, from here $\tilde{U}_k=0$.

Therefore

$$r_{|k-m|}=M\left[\left(U_k-\tilde{U}_k\right)\left(U_m-\tilde{U}_m\right)\right]=M\left[U_k\cdot U_m\right]. \qquad (8)$$

Further, from (3) follows, that at $m\leq(k-N), k\geq N$

$$\begin{aligned}M[u_m g]&=M\left[U_m\sum_{i=0}^{N-1} b_i \xi_{|k-i|}\right]=\\&=\sum_{i=0}^{N-1} b_i M\left[U_m \xi_{|m+N-i+d|}\right]=0,\end{aligned} \qquad (9)$$

As $k=m+N+d$ and for $k\geq m+n, d\geq 0$, from here the least index at $V_{|k-i|}$ is equal

$(m+N-i+d) \ge (m+1+d) > m$, therefore at formation U_m sizes V_j with indexes smaller are used, than at formation g_k. From here follows, that at $n \ge 0$

$$M\left[U_k U_{|k-N-n|}\right] = M\left[U_{|k-N-n|}\sum_{j=1}^{N} a_j U_{|k-j|}\right] + M\left[U_{|k-N-n|} g_k\right] =$$
$$= \sum_{j=1}^{N} a_j M\left[U_{|k-N-n|} U_{|k-j|}\right] = \sum_{j=1}^{N} a_j r_{N+n-j} = r_{N+n}, \tag{10}$$

in particular, at $n = 0,1,2,...,N-1$, we receive the following system of the linear equations for definition of vector
$\mathbf{a} = (a_1, a_2, ..., a_N)$

$$\begin{cases} r_0 a_N & + & r_1 a_{N-1} & + & \dots & + & r_{N-1} a_1 & = & r_N, \\ r_1 a_N & + & r_2 a_{N-1} & + & \cdots & + & r_N a_1 & = & r_{N+1}, \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ r_j a_N & + & r_{J+1} a_{N-1} & + & \cdots & + & r_{N-1+j} a_1 & = & r_{N+j}, \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ r_{N-1} a_N & + & r_N a_{N-1} & + & \cdots & + & r_{2(N-1)} a_1 & = & r_{2N-1}, \end{cases} \tag{11}$$

Or, input correlation matrix $\mathbf{r}_{N,N} = \{C_{ij}\}, i, j = 1,2,\dots,N$ where $C_{ij} = r_{|i+j+2|}$, a vector of free members $\mathbf{r}_N = (r_N, r_{N+1}, \dots, r_{2N-1})$, a vector of unknown parameters $\mathbf{a}_N^* = (a_N, a_{N-1}, \dots, a1)$, system of the equations (11) we will write down in the matrix form

$$\mathbf{r_{NN}} \cdot \mathbf{a_N^*} = \mathbf{r_N^T}, \tag{12}$$

where $(\ )^{\mathrm{T}}$ – a transposing sign. Using Kramer's formulas, we receive the decision

$$a_l = \det \mathbf{r}_{NN}^{(\mathbf{l})} / \det \mathbf{r_{NN}}, \tag{13}$$

where $\mathbf{r}_{\mathbf{NN}}^{(\mathbf{l})}$ – matrix $\mathbf{r_{NN}}$ in which l, the column, is replaced with vector $\mathbf{r_N^T}$.

Now we will define vector $\mathbf{b_N} = (b_0, b_1, \dots, b_{N-1})$. At first we will notice, that

$$\tilde{g}_k = M\left[\sum_{i=0}^{N-1} b_i \xi_{k-1}\right] = \sum_{i=0}^{N-1} b_i M[\xi_{k-1}] = 0. \tag{14}$$

And, designating through $R_n \triangleq M[g_k g_{k-n}]$, we will receive, that at $0 \le n \le N-1$

$$M[g_k g_{k-n}] = R_n = M\left[\left(\sum_{i=0}^{N-1} b_i \xi_{k-i}\right)\left(\sum_{j=0}^{N-1} b_j \xi_{k-n-j}\right)\right] =$$
$$= \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} b_i b_j M[\xi_{k-i}\xi_{k-n-j}] = \sum_{j=0}^{k-n-1} b_j b_{n+j}. \tag{15}$$

From here for definition of vector $\mathbf{b_N}$ we receive system of the nonlinear equations

$$\begin{cases} b_0^2 & + & b_1^2 & + & \cdots & + & b_{N-1}^2 & = & \sum_{j=0}^{N-1} b_j^2 & = & R_0, \\ b_0 b_1 & + & b_1 b_2 & + & \cdots & + & b_{N-2} b_{N-1} & = & \sum_{j=0}^{N-2} b_j b_{j+1} & = & R_1, \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ b_0 b_i & + & b_1 b_{i+1} & + & \cdots & + & b_{N-1-i} b_{N-1} & = & \sum_{j=0}^{N-i-1} b_j b_{j+i} & = & R_i, \\ \cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\ & & & & & & b_0 b_{N-1} & = & R_{N-1}, & & \end{cases} \tag{16}$$

where $R_0, R_1, \dots, R_{N=1}$ – readout of lattice function of correlation of painted normal noise $g_k, R_j = 0$, for $j \ge N$. We will find R_n for $n > 0$. By definition

$$R_n = M[g_k g_{k-n}] =$$
$$= M\left[\left(U_k - \sum_{j=1}^{N} a_j U_{k-j}\right)\left(U_{k-n} - \sum_{i=1}^{N} a_j U_{k-n-i}\right)\right] =$$
$$= M[U_k U_{k-n}] - \sum_{j=1}^{N} a_j M[U_{k-j} U_{k-n}] -$$
$$- \sum_{i=1}^{N} a_i M[U_k U_{k-n-i}] +$$
$$+ \sum_{i=1}^{N}\sum_{j=1}^{N} a_i a_j M[U_{k-j} U_{k-n-i}] =$$
$$= r_n - \sum_{j=1}^{N} a_j r_{|j-n|} - \sum_{i=1}^{N} a_i r_{|i+n|} + \sum_{i=1}^{N}\sum_{j=1}^{N} a_i a_j r_{|n+i-j|} = \tag{17}$$
$$= r_n - \sum_{l=1}^{N} a_l \left(r_{|n-1|} + r_{|n+l|}\right) + \sum_{i=1}^{N}\sum_{j=1}^{N} a_i a_j r_{|n+i-j|}.$$

We receive, that at the assignment of Markov`s discrete normal random process of N th order, with correlation function $\mathbf{r}(nT) = r_n, n = 1,2,\cdots$, (T – a discretization interval), the transfer function of DFF, forming this Markov`s process from normal discrete noise $V_k \sim N(0,1)$, is defined by vectors $\mathbf{a_N}$ and $\mathbf{b_N}$ where vector $\mathbf{a_N}$ is the decision of system of the linear equations, and vector $\mathbf{b_N}$ of a nonlinear equations, kind

$$\begin{cases} \sum_{i=0}^{N-1} r_{i+j} a_{N-i} = r_{N+j}, \quad j = 0,1,\dots,N-1, \\ \sum_{l=0}^{N-1} b_i b_{i+l} = r_l - \sum_{k=1}^{N} a_k \left(r_{|l-k|} + r_{|l+k|}\right) + \\ \qquad \sum_{i=1}^{N}\sum_{j=1}^{N} a_i a_j r_{|l+i-j|}, \quad l = 0,1,\dots,N-1, \end{cases} \tag{18}$$

The system from *2N* equations (18) defines task solution of synthesis of DFF, if as $\mathbf{r}(nT)$ is used normalized correlation function, then $r_0 = 1$ and on exit of DFF, whose coefficients are defined by system (18), the discrete normal noise is forming, representing Marcov`s process of N th order with set correlation function, a zero mean and an single dispersion. DFF has standard structure and can be presented as consecutive connection of two filters: not recursive filter with the final pulse characteristic, defined by vector $\mathbf{b_N}$ forming painted noise $g_k, k = 1,2,\ldots,$ with correlation function $R_0, R_1, \ldots, R_{N-1}$ and $R_n = 0$ at $n \geq N$ (model of a sliding average), and the recursive filter with the infinite pulse characteristic, defined by vector $\mathbf{a_N}$ on which exit demanded random process U_k is formed.

Worded process of synthesis of LDFF as a matter of fact represents a technique of synthesis of channeled LDFF for modelling normal proc esses with the set correlative-spectral characteristics. This technique is generalisation of techniques of synthesis of the similar filters, as here are received expressions for direct calculation of coefficients of the general model of autoregress-sliding of an average , and in the specified literature-only particular cases of synthesis: autoregression or a sliding average (the general method is only declared).

IV. CONCLUSION

The most comprehensible method, allowing to model random processes in real time and without a methodical error, is the method of nonlinear functional transformation of normal randon process, since mathematical expressions of algorithms of modelling, the following from it, allows to parallelize calculations at their realization on multiprocessing COMPUTERS . The basic disadvantage of this method is great size of the preparatory work connected with calculation of parameters of algorithms, thus it is required to make of the decision in process of performance of numerical calculations for separate private problems that does practically impossible automation of process of synthesis of algorithm of modelling.

At modelling of the real processes describing disturbing signal in the bench-imitating environment, it is enough to be limited to resulted casual processes, thus non-stationarity of modelled processes can be considered by changing a mathematical expectation and a dispersion of resulted process. Actually resulted process is stationary in a broad sense, that allows to use a method of forming filters for its modelling.

The method of calculation of coefficients of forming filters developed in work, allows to count coefficients for the general case of process of autoregress-sliding of an average, that minimises number of factors of the differential equation, realizing this filter, and, hence, raises speed of algorithm of modelling. The received closed expressions for coefficients of filters allow to automate process of its synthesis.

REFERENCES

[1] Бажин С.А., Куюмчев Г.В. Программная реализация алгоритма автоматизированного синтеза нелинейного многоканального формирующего фильтра. Раучная сессия ГУАП. Тез. докл. СПб, апрель 2007, с.5

[2] Kuyumchev G.V. Designing of Analog and Digital Forming Filters., International Forum “Information and Communication Technologies: Problems, Perspectives” may 25-30, 2008

[3] Бесекерский В.А. Цифровые автоматические системы. - М.: Наука, 1976. - 576 с.

[4] Изранцев В.В., Шепета Д.А. Моделирование внешних сигналов бортовых приборных комплексов летательных аппаратов пятого поколения // Изв. Вузов / Приборостроение.-2000, №2, с.76-83.

[5] Крамер Г. Математические методы статистики / Пер. с англ; Под ред. А.Н. Колмогорова. - М.: Мир, 1975. - 648 с.

[6] Шалыгин А.С., Палагин Ю.И. Прикладные методы статистического моделирования- Л.: Машиностроение, Л-д. отдел. 1986. - 320 с.

[7] Shepeta D.A. Synthesis of the Non-linear Forming Filters for Radiolocational Systems Input Signal Modeling. Abstracts. ISA Conference St.-Petersburg 2000.